

UNIVERSITE DE GENEVE

FACULTE DES SCIENCES

Département de physique
nucléaire et corpusculaire

Professeur R. Mermod

DETERMINATION EXPERIMENTALE
DU FACTEUR DE FORME
DE LA DESINTEGRATION $K^0 e^3$

THESE

PRESENTEE A LA FACULTE DES SCIENCES

DE L'UNIVERSITE DE GENEVE

POUR OBTENIR LE GRADE DE DOCTEUR ES SCIENCES PHYSIQUES

PAR

Joe MARASCO

de

FAR ROCKAWAY, NEW YORK (ETATS UNIS)

THESE No 1608

GENEVE

Atelier d'impression de la Section de physique

1973

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La faculté des sciences, sur le préavis du
professeur Ronald MERMOD, directeur de thèse, et
d'une commission composée du professeur M. MARTIN
et du Docteur R. TURLAY (CEN - Saclay), autorise
l'impression de la présente thèse, sans exprimer
d'opinion sur les propositions qui y sont énoncées.

GENEVE, le 20 décembre 1972

Le Doyen :

Jean Muller

Thèse No 1608



SOMMAIRE

Dans une expérience électronique faite au Centre d'Etudes Nucléaires de Saclay (France), nous avons observé la désintégration $K_L^0 \rightarrow \pi e \nu$ et mesuré la dépendance en énergie du facteur de forme f_+ du courant hadronique $K-\pi$. Ce facteur de forme ne peut dépendre que d'un seul invariant relativiste, q^2 , le carré de la quadri-impulsion transférée aux deux leptons. En limitant le développement au premier terme, on écrit:

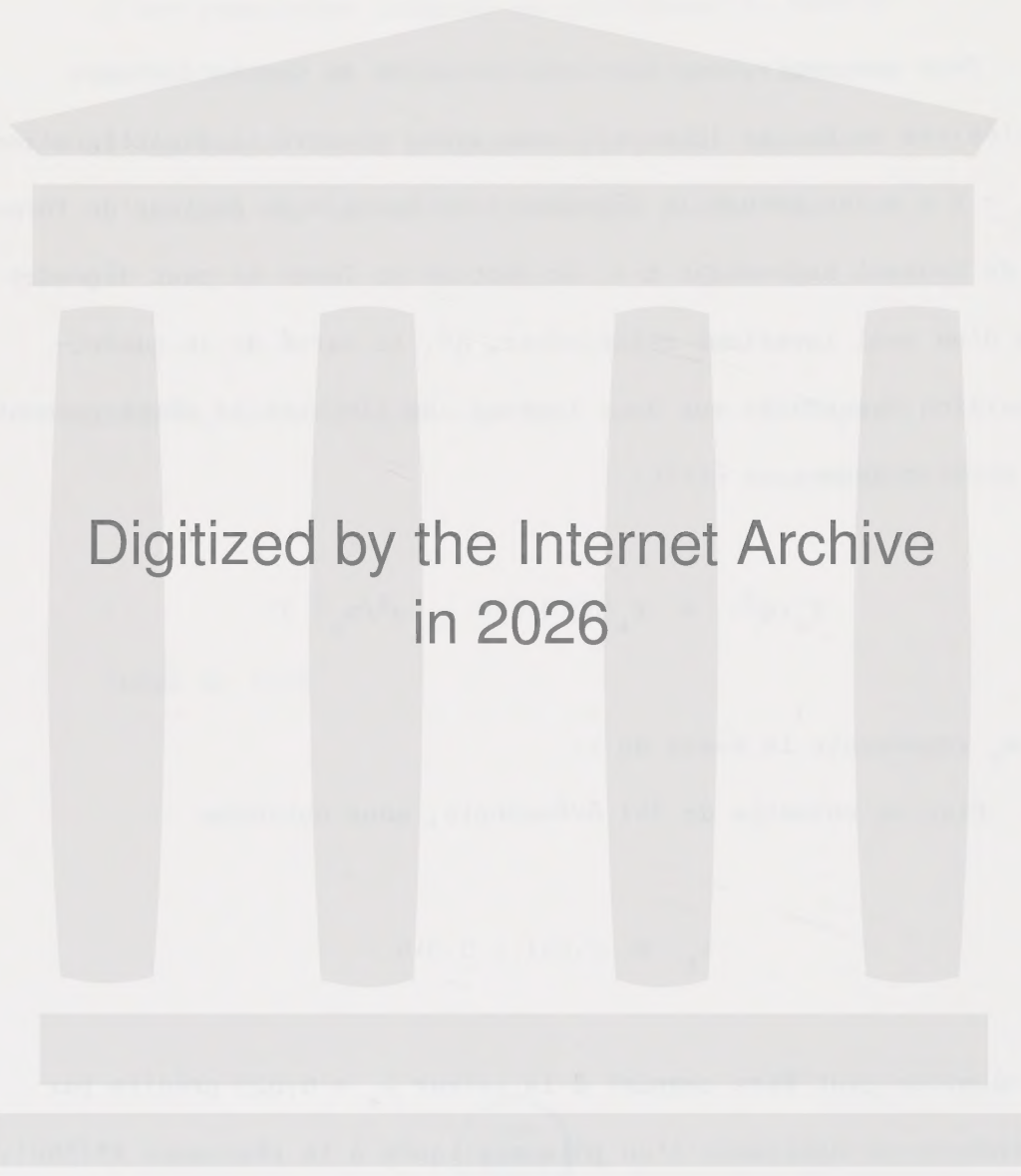
$$f_+(q^2) = f_+(0) (1 + \lambda_+ q^2/m_\pi^2)$$

où m_π représente la masse du π .

Pour un ensemble de 841 événements, nous obtenons

$$\lambda_+ = 0.041 \pm 0.016 .$$

Ce résultat peut être comparé à la valeur $\lambda_+ = 0.023$ prédite par la théorie de dominance d'un pôle appliquée à la résonance $K^*(890)$. Notre analyse n'a mis en évidence aucun terme en q^4 .



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A physicist needs an extraordinary wife, and it is my good fortune to have one. Nothing would be possible without her. I thank her for every minute of our years together.

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I. Introduction.

This thesis is a report of an experiment dealing with one of the semi-leptonic decay modes of the neutral K meson. The K meson complex, consisting of two charged and two neutral mesons, has interested physicists intensely for the past fifteen years; because of its extraordinary richness, the field is far from exhausted and many of the problems are still unresolved. One of these centers on K meson decay, a weak interaction phenomenon, as a probe to study the strong interactions.

The disintegration which permits us to do this is that in which the K meson disintegrates into a pion and a lepton-antilepton pair (an electron or a muon, with the corresponding antineutrino, for example). We can think of this as the creation of a lepton-antilepton pair from the vacuum, accompanied by the transformation of a K meson into a pion. Suppose we express the decay as the product of these two effects; then the knowledge of one enables us to investigate the other. In fact, we can calculate the lepton-antilepton part, a purely weak interaction phenomenon, using the current-times-current theory of Fermi. The K-pion interaction is more complex; the effects of the strong interaction between these two particles cannot be explicitly calculated. We thus try to exploit as much as possible our knowledge of symmetries to minimize the number of parameters needed to describe the interaction. These parameters, called form factors, help us understand the nature of the interaction until the day when a more complete theory relegates them to an historical role.

The disintegrations investigated in the experiment were those involving the long-lived neutral K meson into the final states $\pi^+\mu^+\nu$ ($K\mu 3$) and $\pi^+e^+\nu$ ($Ke 3$). The $K\mu 3$ mode needs two form factors to be completely described. The $Ke 3$ decay needs only one. The simplicity of the latter is particularly enticing; the determination of the form factor dependence on the four-momentum transfer squared suffices to completely determine the entire phenomenon. Moreover, the universality of the weak interactions predicts that one of the two form factors in $K\mu 3$ decay is identical to that found in $Ke 3$ decay. Thus knowledge of the $Ke 3$ form factor gives us an important starting point for the more difficult $K\mu 3$ problem.

The situation in 1970 can be said to have been the calm before the storm. Several experiments seemed to indicate a convergence toward a common value for λ_+ , the coefficient of the linear term in the form factor dependence. In 1971, two high statistics experiments revealed an astonishing lack of agreement: one gave a value of 0.023 ± 0.005 ; ⁽¹⁾ the other yielded 0.05 ± 0.01 . ⁽²⁾ The divergence of these results precludes the usual procedure of taking weighted averages; the averages have no meaning because of the smallness of the error bars on each experiment.

It was suggested in 1970 that with the advent of high statistics experiments it was possible, and perhaps necessary, to investigate the presence of a quadratic term in the development. This requires a detector which preferentially accepts configurations with relatively high four-momentum transfer squared; many events are needed where the quadratic term may manifest itself most clearly.

It was for these reasons that the University of Geneva and the Center for Nuclear Studies at Saclay collaborated on this experiment. A counter and spark chamber spectrometer with high acceptance at high four-momentum transfer squared was used to investigate the questions posed above.

This thesis deals only with the analysis of the Ke3 subsample of the data.

II. Theory.

In this chapter we develop the notations and conventions used throughout the text. We calculate the transition probability for the $Ke3$ disintegration. Finally, we define the parametrization of the form factor, and discuss the value predicted for the parameter involved.

II. a. Matrix Element.

We use the following notations:

- quantities with Greek subscripts or superscripts are four-vectors, i.e. P_K^α ; the index runs from 1 to 4;
- quantities with Latin subscripts or superscripts are three-vectors, i.e. $P_K^i = i^{\text{th}}$ component of $\vec{P}_K$;
- all momenta and energies are in the rest system of the K in this chapter; elsewhere, quantities in the K rest system are denoted by an asterisk (*).

The Dirac matrices are defined as:

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix},$$

$$\gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4.$$

σ_i are the 2x2 Pauli matrices. $i = 1, 2, 3$.

The Dirac equation is

$$(i\gamma_{\mu} P^{\mu} + m)\psi = 0 .$$

The Dirac spinors have the following normalization:

$$\sum_{\text{spin states}} u_{\ell} \bar{u}_{\ell} = \frac{m_{\ell} - i\gamma_{\mu} P_{\ell}^{\mu}}{2m_{\ell}} .$$

The assumptions we make are the following:

- the Hamiltonian can be written in the usual
(current) x (current) way;
- the two component theory of the neutrino is valid;
- the interaction of the lepton fields is local;
- the effects of CP violation are negligible.

We can now write a matrix element for the semi-leptonic disintegration which takes into account both the strong and weak interactions, even though the precise description of the strong interaction is not known. The form factors which appear express our ignorance of the strong interaction. The matrix element is

$$M = \sum_i A_i \left[\bar{u}_{\ell} A_i' (1 + \gamma_5) u_{\nu} \right] .$$

The A_i and the A_i' depend on the type of interaction:

	A_i	A_i'
scalar	f_1	1
vector	$f_2 P_K^\alpha + f_3 P_\pi^\alpha$	γ_α
tensor	$f_4 (P_K^\alpha P_\pi^\beta - P_\pi^\alpha P_K^\beta)$	$\gamma_\alpha \gamma_\beta$

The P^α 's are the four momenta of the K and the π . The form factors themselves are scalar functions of the four-momentum transfer squared. The only invariants are $P_K^\alpha P_{K\alpha}$, $P_\pi^\alpha P_{\pi\alpha}$, and $P_K^\alpha P_{\pi\alpha}$. The first two are constants. The third is related to the four-momentum transfer squared by the following:

$$(P_K^\alpha - P_\pi^\alpha)^2 = -m_K^2 - m_\pi^2 - 2 P_K^\alpha P_{\pi\alpha}.$$

Hereafter we call $-(P_K^\alpha - P_\pi^\alpha)^2$ simply q^2 . In the rest system of the K, the factor $P_K^\alpha P_{\pi\alpha}$ equals $-m_K E_\pi$. Thus the form factors depend only on the energy of the pion in the K rest system. Finally, we note that there are no pseudoscalar or axial-vector terms in the matrix element because the K and the π have the same parity.

If we examine the behavior of M under the time reversal operation, we conclude that time reversal invariance imposes f_2 and f_3 real; f_1 and f_4 are pure imaginary. Redefining a set of f's which are all real, we write the following matrix element:

$$M \propto \bar{u}_\ell \left[f_1' - if_2' P_K^\alpha \gamma_\alpha - if_3' P_\pi^\alpha \gamma_\alpha + if_4' (P_K^\alpha P_\pi^\beta - P_\pi^\alpha P_K^\beta) \sigma_{\alpha\beta} \right] (1+\gamma_5) u_\nu$$

with
$$\sigma_{\alpha\beta} = \frac{1}{2i} (\gamma_\beta \gamma_\alpha - \gamma_\alpha \gamma_\beta) .$$

Redefining the form factors in the following way:

$$f_1' = f_s ,$$

$$f_2' + f_3' = -f_v/m_K ,$$

$$f_3' = -F_v/m_K ,$$

$$f_4' = f_t/m_K^2 ,$$

where the m_K 's are introduced to render the form factors dimensionless, we have

$$\begin{aligned} M \propto f_s \bar{u}_\ell (1+\gamma_5) u_\nu + \frac{if_v}{m_K} P_K^{\alpha-} \gamma_\alpha (1+\gamma_5) u_\nu + \frac{iF_v}{m_K} (P_\pi - P_K)^{\alpha-} \gamma_\alpha (1+\gamma_5) u_\nu \\ + \frac{if_t}{m_K^2} (P_K^\alpha P_\pi^\beta - P_\pi^\alpha P_K^\beta) \bar{u}_\ell \sigma_{\alpha\beta} (1+\gamma_5) u_\nu . \end{aligned}$$

This is the general matrix element for all semi-leptonic decays of the K meson into three particles. There are several simplifications which manifest themselves later. For instance, knowledge that the interaction is of a vector nature allows us to set $f_s = f_t = 0$. In the case of $Ke3$,

all terms in the transition probability which are multiplied by $(m_\ell/m_K)^2$ can be neglected; this is not true for $K\mu 3$. We calculate the general transition probability and at the end set the appropriate terms equal to zero.

We must now calculate $MM^\dagger$. First we simplify M somewhat. Taking the term $\frac{iF}{m_K} \frac{v}{\cdot} (P_\pi - P_K)^\alpha \bar{u}_\ell \gamma_\alpha (1+\gamma_5) u_\nu$, we use the conservation of energy-momentum to write

$$\begin{aligned} \frac{iF}{m_K} v (P_\pi - P_K)^\alpha \bar{u}_\ell \gamma_\alpha (1+\gamma_5) u_\nu &= - \frac{iF}{m_K} v (P_\ell + P_\nu)^\alpha \bar{u}_\ell \gamma_\alpha (1+\gamma_5) u_\nu \\ &= - \frac{F}{m_K} v \left[\bar{u}_\ell i P_\ell^\alpha \gamma_\alpha (1+\gamma_5) u_\nu + \bar{u}_\ell i P_\nu^\alpha \gamma_\alpha (1+\gamma_5) u_\nu \right] \\ &= - \frac{F}{m_K} v \left[\bar{u}_\ell i \gamma_\alpha P_\ell^\alpha (1+\gamma_5) u_\nu + \bar{u}_\ell (1-\gamma_5) i \gamma_\alpha P_\nu^\alpha u_\nu \right] \end{aligned}$$

and with the aid of the Dirac equation,

$$= F \frac{v}{\cdot} \left(\frac{m_\ell}{m_K} \right) \bar{u}_\ell (1+\gamma_5) u_\nu .$$

Turning now to the second and third terms, we observe that in the rest system of the K ,

$$P_K^\alpha \gamma_\alpha = i \gamma_4 m_K ,$$

thus the second term can be written as $-\gamma_4 f \frac{v}{\cdot} \bar{u}_\ell (1+\gamma_5) u_\nu$. The third term can also be simplified

$$P_K^\alpha \gamma_\alpha P_\pi^\beta \gamma_\beta - P_\pi^\alpha \gamma_\alpha P_K^\beta \gamma_\beta = i\gamma_4 m_K P_\pi^\alpha \gamma_\alpha - P_\pi^\alpha \gamma_\alpha i\gamma_4 m_K = im_K (2\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi) .$$

Regrouping these simplifications, we write

$$M \propto \bar{u}_\ell (A - f_v \gamma_4 + iB\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi) (1+\gamma_5) u_\nu$$

where $A = f_s + f_v m_\ell / m_K$ and $B = f_t / m_K$.

Then

$$M^\dagger \propto u_\nu^\dagger (1+\gamma_5) (A^* - f_v^* \gamma_4 - iB^* \vec{\gamma} \cdot \vec{P}_\pi \gamma_4) \gamma_4 u_\ell$$

$$MM^\dagger \propto \bar{u}_\ell (A - f_v \gamma_4 + iB\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi) (1+\gamma_5) u_\nu \bar{u}_\nu (1-\gamma_5) (A^* - f_v^* \gamma_4 - iB^* \vec{\gamma} \cdot \vec{P}_\pi \gamma_4) u_\ell$$

Since the beam is unpolarized and the final polarization of the leptons is not measured, we must sum over all the possible states of spin in the initial and final states. To do this we bring $\bar{u}_\ell$ to the other side of the expression, transforming it to one of calculating traces:

$$MM^\dagger \propto \text{Tr} \left[(A - f_v \gamma_4 + iB\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi) (1+\gamma_5) (-i\gamma_\alpha P_\nu^\alpha) (1-\gamma_5) \times \right. \\ \left. (A^* - f_v^* \gamma_4 - iB^* \vec{\gamma} \cdot \vec{P}_\pi \gamma_4) (m_\ell - i\gamma_\alpha P_\ell^\alpha) \right]$$

and with

$$(1+\gamma_5) (-i\gamma_\alpha P_\nu^\alpha) (1-\gamma_5) = (1+\gamma_5)^2 (-i\gamma_\alpha P_\nu^\alpha) = -2i(1+\gamma_5) (\gamma_\alpha P_\nu^\alpha)$$

we obtain

$$\begin{aligned}
 MM^\dagger \propto -2i \operatorname{Tr} \bigg[& (A\gamma_\lambda P_\nu^\lambda - f_\nu \gamma_4 \gamma_\lambda P_\nu^\lambda + iB\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi \gamma_\lambda P_\nu^\lambda \\
 & + A\gamma_5 \gamma_\lambda P_\nu^\lambda - f_\nu \gamma_4 \gamma_5 \gamma_\lambda P_\nu^\lambda + iB\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi \gamma_5 \gamma_\lambda P_\nu^\lambda) \times \\
 & (A^* m_\ell - f_\nu^* \gamma_4 m_\ell + iB^* \vec{\gamma} \cdot \vec{P}_\pi \gamma_4 m_\ell \\
 & - iA^* \gamma_\lambda P_\ell^\lambda + i f_\nu^* \gamma_4 \gamma_\lambda P_\ell^\lambda + B^* \vec{\gamma} \cdot \vec{P}_\pi \gamma_4 \gamma_\lambda P_\ell^\lambda) \bigg] .
 \end{aligned}$$

Thus there are 36 terms to evaluate. The rules for taking the trace of the various combinations of gamma matrices are found in the literature.⁽³⁾ Remembering that in our metric $P^\lambda = (P^1, P^2, P^3, P^4)$ with $P^4 = iE$, we calculate the first eighteen terms (see table, page 11). The rest of the terms are equal to zero because of the γ_5 factor in the fourth, fifth and sixth term in the first bracket. This follows from

$$\operatorname{Tr} \left[\gamma_5 (\gamma \cdot a) (\gamma \cdot b) (\gamma \cdot c) (\gamma \cdot d) \right] = 4 \det (abcd) .$$

If there are fewer than four gamma matrices, the trace is zero.

Grouping terms, we have

$$\begin{aligned}
 MM^\dagger \propto & -2\operatorname{Re}(Af_\nu^*) m_\ell E_\nu - 2\operatorname{Re}(f_\nu B^*) m_\ell \vec{P}_\nu \cdot \vec{P}_\pi + 2\operatorname{Re}(AB^*) (E_\ell \vec{P}_\nu \cdot \vec{P}_\pi - E_\nu \vec{P}_\pi \cdot \vec{P}_\ell) \\
 & - |A|^2 P_\nu^\alpha (P_\ell)_\alpha + |f_\nu|^2 \left[2E_\nu E_\ell + P_\nu^\alpha (P_\ell)_\alpha \right] \\
 & + |B|^2 \left[(\vec{P}_\nu \cdot \vec{P}_\ell + E_\nu E_\ell) |\vec{P}_\pi|^2 - 2(\vec{P}_\pi \cdot \vec{P}_\ell)(\vec{P}_\pi \cdot \vec{P}_\nu) \right] .
 \end{aligned}$$

<u>constant</u>	<u>first term</u>	<u>second term</u>	<u>value</u>
AA^*m_ℓ	$\gamma_\alpha P_\nu^\alpha$	1	0
$-Af_\nu^*m_\ell$	"	γ_4	$-4iAf_\nu^*m_\ell E_\nu$
iAB^*m_ℓ	"	$\vec{\gamma} \cdot \vec{P}_\pi \gamma_4$	0
$-iAA^*$	"	$\gamma_\beta P_\ell^\beta$	$-4iAA^*P_\nu^\alpha (P_\ell)_\alpha$
iAf_ν^*	"	$\gamma_4 \gamma_\beta P_\ell^\beta$	0
AB^*	"	$\vec{\gamma} \cdot \vec{P}_\pi \gamma_4 \gamma_\beta P_\ell^\beta$	$4iAB^*(E_\ell \vec{P}_\nu \cdot \vec{P}_\pi - E_\nu \vec{P}_\pi \cdot \vec{P}_\ell)$
$-f_\nu^*A^*m_\ell$	$\gamma_4 \gamma_\alpha P_\nu^\alpha$	1	$-4iA^*f_\nu^*m_\ell E_\nu$
$f_\nu f_\nu^*m_\ell$	"	γ_4	0
$-if_\nu^*B^*m_\ell$	"	$\vec{\gamma} \cdot \vec{P}_\pi \gamma_4$	$-4if_\nu^*B^*m_\ell \vec{P}_\nu \cdot \vec{P}_\pi$
$if_\nu^*A^*$	"	$\gamma_\beta P_\ell^\beta$	0
$-if_\nu f_\nu^*$	"	$\gamma_4 \gamma_\beta P_\ell^\beta$	$4if_\nu f_\nu^* \left[2E_\nu E_\ell + P_\nu^\alpha (P_\ell)_\alpha \right]$
$-f_\nu^*B^*$	"	$\vec{\gamma} \cdot \vec{P}_\pi \gamma_4 \gamma_\beta P_\ell^\beta$	0
iBA^*m_ℓ	$\gamma_4 \vec{\gamma} \cdot \vec{P}_\pi \gamma_\alpha P_\nu^\alpha$	1	0
$-iBf_\nu^*m_\ell$	"	γ_4	$-4iBf_\nu^*m_\ell \vec{P}_\pi \cdot \vec{P}_\nu$
$-BB^*m_\ell$	"	$\vec{\gamma} \cdot \vec{P}_\pi \gamma_4$	0
BA^*	"	$\gamma_\beta P_\ell^\beta$	$4iBA^* \left[E_\ell (\vec{P}_\pi \cdot \vec{P}_\nu) - E_\nu (\vec{P}_\pi \cdot \vec{P}_\ell) \right]$
$-Bf_\nu^*$	"	$\gamma_4 \gamma_\beta P_\ell^\beta$	0
iBB^*	"	$\vec{\gamma} \cdot \vec{P}_\pi \gamma_4 \gamma_\beta P_\ell^\beta$	$4iBB^* \left[\vec{P}_\pi ^2 (\vec{P}_\nu \cdot \vec{P}_\ell + E_\nu E_\ell) - \right.$ $\left. 2(\vec{P}_\pi \cdot \vec{P}_\ell)(\vec{P}_\pi \cdot \vec{P}_\nu) \right]$

II. b. Ke3 Transition Probability.

We now write the transition probability. This is the matrix element squared multiplied by a phase space factor. The phase space factor is the conservation of energy-momentum for the entire process. There are three particles in the final state. We must integrate

$$dI = \frac{d\vec{P}_\pi}{E_\pi} \frac{d\vec{P}_\ell}{E_\ell} \frac{d\vec{P}_\nu}{E_\nu} \delta(\vec{P}_\pi + \vec{P}_\ell + \vec{P}_\nu) \delta(m_K - E_\pi - E_\ell - E_\nu),$$

over all the variables except two. The choice of the two excluded variables is ours. For instance, if we choose E_π and E_ℓ as the two variables, it can easily be shown that the phase space integration yields a constant; the distribution of events in phase space is uniform.⁽⁴⁾ By looking at the departure from uniform density in this system of coordinates, the physics can be discerned.

Another possible pair of variables is the following: $|\vec{P}_\pi|$, the module of the pion momentum in the rest system of the K, and $\cos \theta$, where θ is the angle between the pion and the neutrino in the same system. With these variables, the phase space integral is

$$I(|\vec{P}_\pi|, \cos \theta) d|\vec{P}_\pi| d(\cos \theta) = \frac{|\vec{P}_\pi|^2}{E_\pi} \frac{E^2 - |\vec{P}_\pi|^2 - m_\ell^2}{(E + |\vec{P}_\pi| \cos \theta)^2},$$

where $E = m_K - E_\pi$.

Expressing the matrix element squared in these variables, we now write the expression for the transition probability:

$$W(|\vec{P}_\pi|, \cos \theta) = \underbrace{\frac{|\vec{P}_\pi|^2}{E_\pi} \frac{(E^2 - |\vec{P}_\pi|^2 - m_\ell^2)^2}{(E + |\vec{P}_\pi| \cos \theta)^4}}_{I'} \times$$

$$\left\{ -2\text{Re}(A f_V^*) m_\ell \left[E + |\vec{P}_\pi| \cos \theta \right] - 2\text{Re}(f_V B^*) m_\ell \left[E + |\vec{P}_\pi| \cos \theta \right] |\vec{P}_\pi| \cos \theta \right.$$

$$+ 2\text{Re}(A B^*) |\vec{P}_\pi| \left[E + |\vec{P}_\pi| \cos \theta \right] \left[E \cos \theta + |\vec{P}_\pi| \right] + |A|^2 \left[E + |\vec{P}_\pi| \cos \theta \right]^2$$

$$\left. + |f_V|^2 \left[|\vec{P}_\pi|^2 \sin^2 \theta + m_\ell^2 \right] + |B|^2 |\vec{P}_\pi|^2 \left[(|\vec{P}_\pi| + E \cos \theta)^2 + m_\ell^2 \sin^2 \theta \right] \right\}$$

The $\text{Ke}3$ disintegration is usually taken to be purely vector.

The following upper limits have been established for the scalar and tensor parts:

$$f_s/f_V < 0.07 ,$$

$$f_t/f_V < 0.35 ,$$

at the 68% confidence level.⁽⁵⁾

With f_s and f_t equal to zero, all terms multiplied by B disappear, and A becomes $F_V m_\ell/m_K$. The transition probability can then be written as:

$$W \propto \left[(m_\ell/m_K)^2 |F_V (E + |\vec{P}_\pi| \cos \theta) - f_V m_K|^2 + |f_V|^2 |\vec{P}_\pi|^2 \sin^2 \theta \right] \times I'.$$

For $\text{Ke}3$ decay the first term is negligible because of the factor

$(m_\ell/m_K)^2$. The entire transition probability thus depends on only one form factor, f_V .

II. c. Parametrization of the Form Factor.

A more convenient parametrization comes from writing the vector matrix element in the form:

$$M \propto (i/m_K) \left[f_+ (P_K^\alpha + P_\pi^\alpha) + f_- (P_K^\alpha - P_\pi^\alpha) \right] \bar{u}_\ell \gamma_\alpha (1 + \gamma_5) u_\nu ,$$

where f_+ and f_- are linear combinations of the previously defined f_v and F_v . A similar analysis shows that the form factor f_- can be neglected because it is multiplied by $(m_\ell/m_K)^2$ in the expression for the transition probability. The transition probability for $Ke3$ becomes

$$W(E_e, E_\pi) dE_e dE_\pi \propto |f_+|^2 \left[(E_\pi - A)m_K + 2E_e E_\nu \right] ,$$

where A is the maximum allowable energy for the pion, given by:

$$A = \frac{m_K^2 + m_\pi^2 - m_e^2}{2m_K} .$$

As we have shown, f_+ can only depend on q^2 , where $q^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$. Now we investigate the variation of f_+ with q^2 .

The simplest assumed variation is linear:

$$f_+(q^2) = f_+(0) (1 + \lambda_+ q^2 / m_\pi^2) .$$

It can be shown using dispersion relation techniques⁽⁶⁾ that if the

disintegration is mediated by the exchange of a $K\pi$ resonance, we have:

$$\text{for an s wave resonance : } f_+(q^2) = f_+(0) ,$$

$$\text{for a p wave resonance : } f_+(q^2) = f_+(0) \frac{M^2}{M^2 - q^2} ,$$

where M is the mass of the intermediate state.

For the p wave resonance:

$$f_+(q^2) = f_+(0) \frac{1}{1 - q^2/M^2} .$$

(q^2/M^2) is small, therefore

$$\frac{1}{1 - q^2/M^2} \approx 1 + q^2/M^2 ,$$

which yields

$$\frac{\lambda_+}{m_\pi^2} = \frac{1}{M^2} ,$$

or

$$\lambda_+ = m_\pi^2/M^2 .$$

If the disintegration is mediated by the $K^*(890)$, we have

$$\lambda_+ = 0.023 .$$

The relation between λ_+ in $K^0 e3$ decay to that in $K^+ e3$ decay can be expressed easily if the $|\Delta I| = \frac{1}{2}$ rule is valid. The decay

rates are written :

$$\Gamma(K_2^0) = \left| \sqrt{\frac{2}{3}} A\left(\frac{1}{2}, \frac{1}{2}\right) + \sqrt{\frac{1}{3}} A\left(\frac{3}{2}, \frac{1}{2}\right) - A\left(\frac{3}{2}, \frac{3}{2}\right) \right|^2 ,$$

$$\Gamma(K^+) = \left| \sqrt{\frac{1}{3}} A\left(\frac{1}{2}, \frac{1}{2}\right) - A\left(\frac{3}{2}, \frac{1}{2}\right) \right|^2 ,$$

where $A(I, I_3)$ is the amplitude with isospin I and isospin z -component I_3 . The $|\Delta I| = \frac{1}{2}$ rule implies that $A(\frac{3}{2}, \frac{1}{2}) = A(\frac{3}{2}, \frac{3}{2}) = 0$. Thus the K_L^0 rate should be twice that of the K^+ . The variation with q^2 should be identical: $\lambda_+(K^0 e3) = \lambda_+(K^+ e3)$.

III. Experimental Arrangement and Data-taking.

We divide this chapter into three parts: the beam, the spectrometer, and the electronics.

III. a. The Beam.

The experiment was performed at the accelerator Saturne. A 1 x 1 x 2 cm uranium target introduced into the flux of 3 GeV/c protons created the K_L^0 beam. Typical machine conditions produced a spill with a 200 millisecond flat-top, a burst coming every 3.3 seconds.

The K_L^0 beam, with its attendant neutrons and gamma rays, travelled to the experimental zone via the beam canal shown in figure 1. The canal at 30° went to the experimental zone; the canal at 90° was used to monitor the beam production. To convert the gamma rays, two lead bricks of 2.5 cm thickness were used. The two sweeping magnets, Zephyr and Minus, operated at maximum current to eliminate the charged particles. A series of collimators defined the beam and kept down the background. The distance from the target to the center of the spectrometer magnet measured 12.1 meters, from the target to the beginning of the disintegration zone about eight meters.

Figure 2 shows the production cross-sections measured by Smith and Piroué⁽⁷⁾ under similar conditions (protons at 3 GeV/c and beam angle of 30°). We make the reasonable hypothesis that the production cross-section for K^0 is similar to that of the K^+ , that the neutron

production cross-section is similar to that for protons. Two points are worth noting: the K production cross-section is known in a relatively limited momentum region, the tail of the distribution being quite poorly known; the neutron flux varies from 100 to 1000 times the K flux at a given momentum. This constituted the fundamental background problem of the experiment.

III. b. The Spectrometer.

The spectrometer used in this experiment was a modified version of the spectrometer used in an earlier experiment on Ke^4 decays.⁽⁸⁾ Certain parts of the old set-up were replaced or improved. Others were left unchanged. Figure 3 shows a plan view of the final experimental arrangement, an arrangement which evolved during the course of the data taking. The results given in this thesis are based on events which were collected with the apparatus in its final state.

A helium-filled bag, extending from inside the last collimator to the first hodoscope, defined the decay zone and diminished the multiple scattering. The first element encountered by the secondaries was the hodoscope H1, a two-plane array of scintillators arranged with the vertical staves in the first plane and the horizontal staves in the second plane. Figure 4a shows the relative sizes of the different scintillators. The narrow scintillators in the central region were connected to Radiotechnique 20-10 photomultipliers. The wider scintillators were connected to standard 56DVP photomultipliers.

Immediately behind H1 was a large plate used to shield the photomultipliers from the stray field of the analyzing magnet. The plate, composed of a sandwich of two iron plates, was welded at several seams because of its size. Needless to say, a rectangular hole the size of the "window" of H1 let the secondaries through.

Two wire spark chambers were placed between the shielding plate and the front face of the analyzing magnet. Each chamber consisted of two gaps and each gap furnished data in both the horizontal and vertical projections. These four gaps determined the particles' trajectories entering the magnet. We called these chambers CF1 and CF2; their dimensions were 900 x 300 mm and 1700 x 560 mm respectively.

Since the chambers used in this experiment have been described in extenso elsewhere,⁽⁹⁾ we will limit our description. The readout system was magnetostrictive; the precision of the spark positions was approximately 0.8 mm. A system of fiducial marks (certain wires which could be pulsed) allowed measurement of the propagation velocity of the magnetostrictive signals. Runs using no magnetic field allowed us to align the chambers by software, using straight-through tracks in which a large number of gaps responded. Finally, we determined the relative efficiency of the chambers under experimental conditions. This was done completely by software and is described later.

The spectrometer magnet Albatross had a central field of 5 kilogauss and the following gap dimensions - width: 150 cm,

height: 40 cm, depth: 20 cm. The integral of $\vec{B} \cdot d\vec{l}$ was about 10^5 gauss-centimeters. The spectrometer magnet presented a barrier to charged particles of less than approximately 60 MeV/c. A Hall Effect probe monitored the field.

Three chambers, each containing two gaps, localized the trajectories after the magnet. We called them CF3, CF4 and CF5, respectively. Only chambers three and five were used to reconstruct the tracks, the fourth chamber, inclined 15° , being used to resolve the horizontal-vertical ambiguity. The chamber dimensions were - CF3: 1700 x 560 mm, CF4 and CF5: 2000 x 1000 mm. These chambers functioned in a like manner to CF1 and CF2. In summary then, the information from the eight useful gaps, four before the magnet and four after, determined the deviation of the particles in the magnetic field, and hence their momenta. The chambers before the magnet served to reconstruct the decay vertex as well.

In between CF3 and CF4, the secondaries encountered a second hodoscope, H3. (H3 was in front of H2 for historical reasons; it was the last element to be added to the detector.) In H3, eight scintillators were arranged in the configuration shown in figure 5. They were connected to 56DVP photomultipliers.

Directly behind the last wire chamber a second large plate shielded the photomultipliers of the Cerenkov counter from the magnetic field. This large, gas Cerenkov counter was used to detect the presence of an eventual electron. The counter was filled with butane at atmospheric pressure. Figure 6 shows in schematic form the side view of the counter. The optical system of this counter

was designed so that the focal plane was at the center of the magnet. In certain cases this arrangement enabled us to determine which secondary had triggered the counter. The Cerenkov light was generated between the entry window and the first set of mirrors, labelled M1. In the shortest distance at least 80 photons were produced. M1 consisted of four mirrors. Each was individually adjustable on its supports. The ensemble of mirrors formed a spherical cross section. The Cerenkov light was then reflected to the mirror M2; this mirror, also of spherical cross section, brought the light to the photomultipliers.

The "box" which contained the photomultipliers was completely self-contained and could be removed from the body of the Cerenkov counter. By removing the photomultipliers and tracing the outlines of their faces onto the plexiglas plate we were able to map, using a laser, the entire trajectory of any track. This laser technique was used to adjust the mirrors and to ensure proper alignment. The photomultipliers used for the counter were 56DVP's and 58DVP's. Figure 7 shows the arrangement of photomultipliers. In certain cases the ring of Cerenkov light was shared by two or more photomultipliers. Given the ring size, the size of the photomultiplier faces and their placement, the probability that not enough light will be received by any of the photomultipliers can be determined. We should mention that the 58DVP's have a rather poor efficiency near the edges. Unfortunately our optical system used the whole face of the photomultiplier. A calibration performed at the linear accelerator at Orsay gave a global efficiency (averaged over all angles) of 50%,

a rough number confirmed in the experiment. Naturally, the calculation of this counter's efficiency, in the local sense, is of capital importance to the analysis.

The next element in the detector system and the last to participate in the trigger is the hodoscope H2. It is similar to H1, two planes of scintillators. The first contained the horizontal staves and the second the vertical staves. Figure 4b shows the relative sizes of the scintillators. The hodoscope is divided into two halves, right and left, in both the horizontal and vertical planes. The rectangular hole in H2 was a vestigial remain from the Ke^4 experiment, where it served to let through the K^+ beam. 56DVP's were used.

A series of optical spark chambers completed the detector. First was a series of three shower chambers, each of eight gaps. The nine plates of each module each measured 2 mm thick. Each module thus accounted for one radiation length. Lead plates, 6 mm thick, were located between the modules. Each of these plates represented one radiation length. Thus the ensemble totaled five radiation lengths. These chambers aided in the identification of the electrons by their characteristic showers. They provided a check of the Cerenkov counter in the analysis.

Behind the shower chambers was a large range chamber of 32 gaps. This chamber was composed of thick copper and brass plates. The thickness of these plates was calculated to differentiate between the pion and muon in $\text{K}\mu^3$ decay. The Ke^3 analysis does not depend on this chamber, and no further description is offered.

After several reflections through a mirror system, the information from the optical spark chambers was recorded by a Flight Research camera. A standard run was taken to be a full roll of 35 mm Ilford Mark II film or about 3200 events.

III. c. Electronics and Trigger Logic.

The trigger logic fired the wire and optical spark chambers when a K meson disintegrated and the two charged secondaries traversed the entire spectrometer. The basic logic modules were retained from the Ke4 experiment, where the presence of the charged K enabled us to put into coincidence with an "entering K" signal all the other units in the detector. In this experiment we worked with a neutral K, so this signal had to be fabricated in some other way.

The basis for the multiple coincidence is a single two-fold coincidence, hereafter called the pilot signal because it initiated the entire logic cycle. One possible way to fabricate the pilot is to require a coincidence between any counter in the horizontal plane in H1 with any counter in the vertical plane in H1. A two-particle trigger is justified by a sufficient number of counters in H1 and H2 in coincidence with the pilot signal. This method proved unsatisfactory; the ratio of two-particle triggers to pilot signals was too low. This can be understood from the neutron background. Frequently a neutron interacted in the first plane of H1, producing a recoil proton which triggered a counter

in the second plane. If the scintillator in the first plane was also triggered, a pilot signal was produced; if not, any other accidental in any counter of the first plane sufficed to fabricate a pilot signal. Of course, only a small fraction of these pilot signals were converted into two-particle triggers after the examination of the other scintillators, but each pilot signal represents a certain dead time, so it was advantageous to diminish the pilot to trigger ratio.

This was done by capitalizing on the low momentum of many of the recoil protons. They did not pass the barrier introduced by the spectrometer magnet. A pilot signal was generated each time there was a coincidence between any signal in the vertical plane of H1 (the upstream plane) and any signal in the horizontal plane of H2 (also the upstream plane). At this point we knew at least one particle had passed through the magnet. If we had a signal in a counter of H1-vertical at time t_0 , we produced a pilot signal if a counter responded in H2-horizontal at time $t_0 + 10$ nanoseconds. To validate the rest of the trigger, we demanded in each of the two planes of H1 two counters within six nanoseconds of each other (one of the vertical plane's counters has taken part in the pilot fabrication; obviously all four must be in coincidence with the pilot) and in H2 two counters in each plane within sixteen nanoseconds of each other (here again one of the horizontal plane's counters has taken part in the pilot fabrication and all are in coincidence with the pilot). We established these limits on the basis of jitters in the photomultiplier response and the fast logic

circuits, as well as a consideration of the time of flight differences between an ultrarelativistic electron and a "slow" pion. As the pilot was always fabricated by the faster particle (it got from the decay vertex to H1 first), we had to be careful not to discriminate against combinations in which the slower particle did not reach H2 within the limit of the coincidence.

Further sophistication proved necessary to reduce the background problems. The hodoscope H3 was added to tighten the coincidence requirements; we required two more counters in coincidence with the pilot. With the addition of H3, we worked with an overall ten-fold coincidence (2x2 in H1, 2x2 in H2, and 2 in H3). The effect was considerable, reducing the trigger rate by 40%.

The geometry of our apparatus enabled us to eliminate another type of false trigger at the level of the electronics. H2 is divided into two halves, left and right, in both the horizontal and vertical planes. The table below shows the combinations of counts in the two planes:

horizontal		vertical		
left	right	left	right	
1	1	1	1	permitted
2	0	2	0	permitted
0	2	0	2	permitted
1	1	2	0	impossible
0	2	1	1	impossible
0	2	2	0	impossible
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Clearly there are more "impossible" combinations, those in the table being given as examples. A fast logic circuit vetoed events with inadmissible combinations.

The signals from the 14 photomultipliers of the Cerenkov counter were divided into 5 groups. See figure 7. The grouping maintained some of the localization properties of the optical system.

Figure 8 shows the entire logic. All the information was brought to a fast gate by a trigger mode circuit, which enabled us to choose the conditions of the trigger. For example, we could require $H1:2x2 + H2:2x2$. To that we could add 2 scintillators from H3. The Cerenkov counter could be put in coincidence or anti-coincidence or left out of the trigger entirely. All information from the logic system was stored on magnetic tape, whether or not it was used in the trigger.

The time sequence was as follows. An event was recognized by the fast logic and the order was given to the master spark gap to fire the chambers. During this time the on-line computer, a CII 90-10, dumped the information from the previous event from its buffer locations onto magnetic tape. It then "interrogated" the logic units about the scintillator information memorized when the coincidence was fabricated. This information, along with the spark chamber data was then put into the appropriate buffer locations. The cumulative distributions stored by the computer permitted a large number of systematic checks. A cathode ray tube enabled us to visualize the spark chamber information and to reconstruct some of the events by eye. We verified the optical spark chamber performance with a television camera installed in the zone.

The experimental trigger was a 10-fold coincidence, with the Cerenkov counter as a "spectator". In this way we accumulated Ke3 and $\text{K}\mu 3$ events simultaneously. The eventual separation was made off-line when the optical spark chamber photographs could be examined.

IV. Data Reduction and Event Selection.

Between January and July 1971 we acquired about 200,000 events on magnetic tape and on film. Of these, only about 25% were events originating in the decay of a K_L^0 . Among the remaining events, a significant portion was lost during the reconstruction phase. Finally, to filter out events which simulated $Ke3$ decays, cuts were made on the data. We divide this chapter into three parts: the event reconstruction, the possible contaminations, and the cuts applied to obtain the final reduced sample.

IV. a. Event Reconstruction.

Certain technical problems, revealed in the analysis of early runs, determined the data reduction chain. One problem was the high incidence of parasitic tracks from the neutron flux. In fact it was impossible to proceed as usual with a preliminary scan of the films to eliminate background events. Too few events would have been eliminated. We decided to start with the information recorded on magnetic tape, using the films as a later check.

The Cerenkov counter's efficiency determined the next important decision. There were two procedures open to us. The first would process the entire sample of 200,000 events systematically, without treating the various modes differently. All $Ke3$ events would have an equal chance of being selected whether or not they triggered the Cerenkov counter. A scan of the corresponding photograph for an

electron track in the optical chambers would signal a possible Ke3 . There are two disadvantages. The entire sample must be processed. Electrons of less than 250 MeV/c tend not to shower; the scanners must count sparks to distinguish between low momentum electrons and pions or muons. Our chambers did not function optimally. We believed it almost impossible to avoid a bias from this effect.

In the second method, only events which triggered the Cerenkov counter would be considered. This is costly: we lose about half of our events due to the inherent inefficiency of the counter. Another disadvantage is the need to calibrate the efficiency (in the local sense) of this counter so as to eliminate biases. There are, however, several advantages to this method. Only about 9200 events out of the total sample need to be examined. This would enable us to learn relatively quickly what the problems of the analysis (also applicable to $\text{K}\mu 3$) would be. A second advantage: we limit contamination to only those modes which trigger the Cerenkov. That this drastically reduces the contamination is shown later.

We decided to use a hybrid of methods one and two. Method two was employed to isolate a sample of Ke3 (Cerenkov) events; this sample serves as the basis of this thesis. It was foreseen that in the analysis of the remaining triggers, a second sample of Ke3 events could be isolated by the systematic scanning. This work is still in progress and we shall have nothing more to say about it.

The first step in the data reduction was to select those events which had a Cerenkov signal and to reconstruct such events. We now describe in some detail the program, called Artemis, used to

reconstruct the events from the spark chamber data. The program was specially adapted to our requirements from the program used in the Ke4 experiment.

The original tape contained the wire chamber information in "coups spadac" or arbitrary units derived from the Spadac system 20 Mhz clock.⁽¹⁰⁾ In addition, it contained in coded form the information as to the triggered scintillators in each hodoscope, as well as the triggered Cerenkov group. Each gap of the wire chambers gave signals for the two projections; we were able to record as many as six sparks per projection per gap. The first step of the program transformed the "coups spadac" (temporal measure of the sparks) into positions in space. We eliminated during this stage all the parasitic sparks which fell outside the physical limits of the chambers.

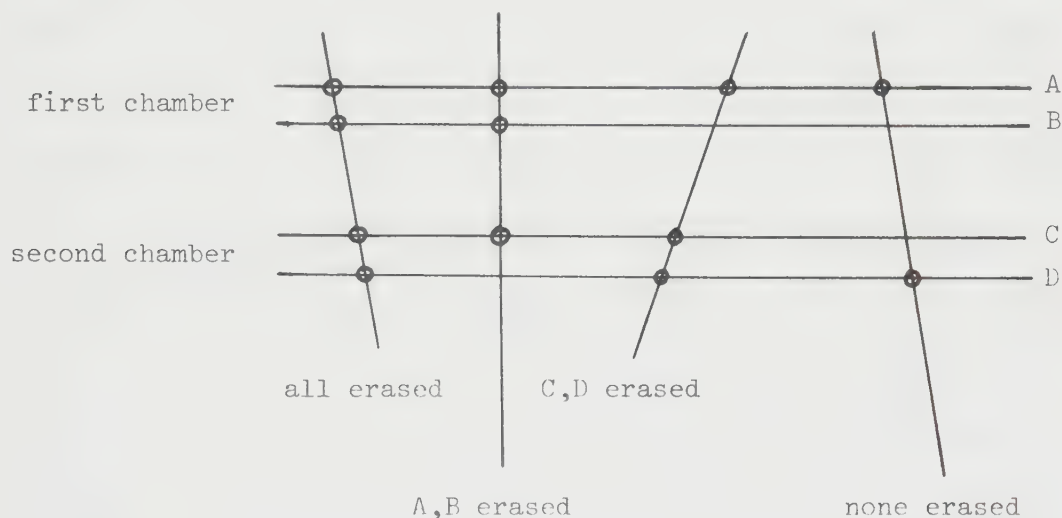
We worked by the method of projections, that is, the horizontal and vertical tracks were reconstructed individually. The method was similar for the two projections. A final step associated the tracks in one projection with those found in the other.

We define here the term "fragment". The chamber data was divided into two groups, one before and one after the magnet. A fragment is a half-track in a given projection. During the first phase of the reconstruction, we tried to reconstruct all the possible fragments using the four gaps, each one having between zero and six sparks. The reconstruction followed a certain order: fragments with a maximum number of sparks were reconstructed first. In a table of the fragments for each quadrant (quadrant refers to the horizontal

or vertical projection, before or after the magnet), we first found those fragments reconstructed using four sparks, then those with three sparks, then those with two. Finally, we reconstructed even those fragments consisting of only two sparks in gaps in the same chamber. In this case the two x-coordinates are very close; the smallness of the lever arm leads to large errors. We reconstructed such fragments anyway, in order to conserve events which might otherwise fail.

The detailed process will not be described here, but the interested reader can refer to the works of Zanella⁽¹¹⁾ and Suter.⁽¹²⁾ In all cases, the fragment candidates had to fulfill certain conditions before they were accepted. Among these conditions we mention the following. If the fragment is before the magnet, it must pass through the disintegration zone. It must have a slope and an ordinate (position at the center of the magnet) within certain limits; this condition helps eliminate tracks which are too inclined, as well as those which hit a pole of the magnet. The chi-squared of the adjusted line must not exceed certain limits in order to eliminate tracks which are doubtful or scattered; the limits on the slopes, ordinates and chi-squareds, different for each quadrant, were set after a Monte Carlo study. The fragment must pass through a triggered scintillator within certain tolerances. This condition was optional in the beginning, but because of a relatively high false trigger rate, we were obliged to include it in the automatic version of the program. The number of good events lost was small. A significant amount of calculation time was gained by reducing the number of possible fragments.

Good events would have been lost had a spark been used first in a wrong track and then been unavailable for reconstructing the correct one. Therefore, the program did not automatically erase a spark once used in a fragment. This concept had to be modified because of the great number of combinations of sparks in the four gaps. A procedure which worked to reduce the number of combinations erased the sparks which were found in the same chamber for fragments reconstructed from three or four sparks:



We stopped the construction at 20 fragments per quadrant. Clean events had many fewer, often only two or three.

The next phase paired the fragments, in each projection, before and after the magnet. All combinations were tried. In permitted combinations the difference between the positions of the fragments at the center of the magnet was within certain limits. This difference was a function of the momentum of the particles. To avoid a bias, we calculated the approximate momentum of each association candidate and

compared its difference to the permitted difference for Monte Carlo events of the same momentum. Here the cuts were not very strict. At the completion of this stage fragments in each projection had been paired into candidate tracks. Now we selected the two tracks in each projection to make up the final event.

The selection of the two tracks was done in the following way. We established in decending order of importance, a hierarchy of conditions which the event had to satisfy. At each step, certain combinations were eliminated. At the end of this process the "best" event, and we hope the correct one, survived. The criteria were the following: (a) the two particles had to have opposite charge; (b) the sum of the momenta of the two particles had to be greater than 200 MeV/c and less than 2400 MeV/c; (c) within a certain margin for error, the decay vertex of the two traces had to be upstream of H1. If no combination of two traces satisfied the above conditions, the event failed in the automatic version of the program. If at this point there were still several combinations which were considered acceptable, the hierarchy continued: (d) preference was given to the combination which had the most triggered scintillators touched by reconstructed tracks; (e) preference was given to the combination with the greatest number of total sparks in the reconstructed tracks. Condition (e) was applied only where equality after (d) existed. In reconstructing the vertical projection (done last) the vertex found in the horizontal could be used to select the best combination in the vertical. The agreement or lack of it proved a powerful tool in selecting the best combination.

Finally, using the information furnished by the scintillators, we associated the tracks found in the two projections. In a small number of configurations, a remaining ambiguity was resolved by associating the corresponding tracks in the optical chambers.

During the early stages of the analysis we discovered the loss of many events caused by a lack of sparks in the chambers rather than a malfunction of the reconstruction program. We added a subprogram enabling us to recover events with fewer than the required number of sparks in the vertical projection after the magnet. This could be done because this quadrant is of little importance in the determination of the particle's momentum and of no importance in determining the angles at the decay vertex. We recovered several other types of marginal events with this subprogram.

We had several options in using Artemis, both as to the functioning of the program and as to the printout of the results. For a systematic study, we could request the printout of every event, successful or not. The printout could be varied to include any or all of the following: a table of unsorted sparks; a table of fragments for each projection; a graphic display of the sparks in each projection, coded for their association into tracks; and finally, in the case of a completely successful event, the entire kinematics of the event for the various mass assignments for each of the particles. This complete output was extremely useful for debugging and for detailed study of the events. Its length, 300 lines of computer output per event, prohibited its use in the later phases of the analysis. For a more rapid study the program, switched into automatic mode,

printed out only successfully reconstructed events. The rest were "lost". This saved considerable computation time since the program stopped the processing of an event when the horizontal projection failed.

The program was tested with a sample of events which had been reconstructed "by hand". The efficiency, based on those events which could be reconstructed, was about 90%. On a CDC 6600 computer the computation time, depending on the version, varied between 400 and 800 milliseconds per event.

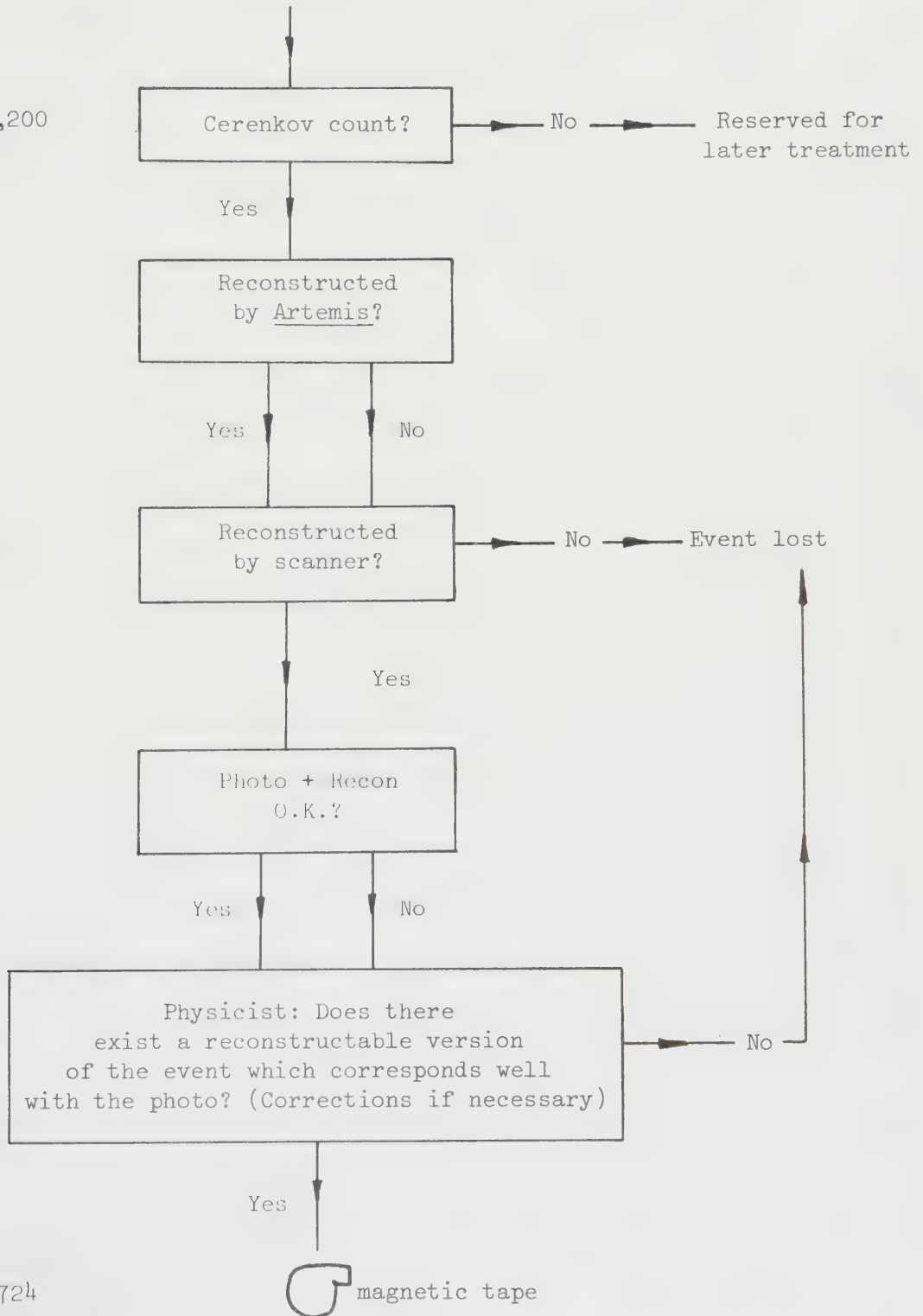
We started the collection of the Ke3 event sample before the reconstruction program was in its final form. A manual check was made to insure that the program was not eliminating too many events. We demanded the complete output, as described above, for all events, completely reconstructed or not. We then used scanning personnel to check each event to see if any were being lost. All reconstructed events were then checked by a physicist, his primary concern being the correctness of the reconstruction. The photographs were scanned and the traces matched with the wire chamber data. Errors in association between the horizontal and vertical projections were corrected at this stage. The physicist also verified the coding of the optical chamber information and the overall consistency of the event; the photo had to indicate at the very least the possibility of an electron. The events which survived the entire filter were then put on magnetic tape for further analysis. The entire chain is shown on the following page.

At the end of this filtering process, we had collected a total of 2724 events, all presumed to be either Ke3 events or events with two electrons. The presence of these events is explained in the next section.

200,000

All Events

9,200



2724

IV. b. Contamination.

We examined the working sample of 2724 events to determine the possible contamination from non-Ke3 events which trigger the detector, as well as Ke3 events which are reconstructed with insufficient precision.

The contamination from other modes divides into two classes: those with electrons and those without electrons. We examine the second class first.

Events in our sample must fulfill the following conditions, as described in chapter 3: two and only two particles must be detected in three hodoscopes and at least one group of photomultipliers in the Cerenkov counter must be triggered.

Therefore, for an event without an electron, we must have had two synchronous tracks, one of which produced a delta ray in the forward wall of the Cerenkov counter (or further upstream) but did not continue on to trigger H2. Events of two tracks plus a third track, an electron, were eliminated by the trigger condition of two and only two counters in each hodoscope. The contamination due to these events (delta ray production) was calculated to be less than 1%.

Contamination from events with electrons further divides into two classes: those with two electrons and those with one electron. In order to monitor events which come from two electrons, we carried along in the Ke3 event selection chain events which were identified as two electron events on the basis of the photographs. In general, these events resulted from the Dalitz decay of a pi zero ($\pi^0 \rightarrow \gamma e^+ e^-$).

The pi zero could have come from a neutron interaction in the helium (for example: $n p \rightarrow n p \pi^0$). Since the angle between the two electrons was usually very small, a certain proportion of these events failed during the reconstruction phase; the two tracks before the magnet being indistinguishable. During the scanning of the films, two electron events were noted, and included in the final sample of Ke3 candidates. Of 2724 events, 340 were identified as two electron events on the basis of the photographs. These two electron events, kinematically analyzed as Ke3 events, fell almost exclusively along the lower left edge of the Dalitz plot and thus constituted a possible source of bias. These 340 events were used to establish kinematic cuts on the remainder of the sample to eliminate events in which two electrons were present but not explicitly identified in the photographs. In general one of the electrons was of low momentum and failed to shower in the shower chambers. We found the efficiency of the kinematic cuts on the identified two electron events to be 83%. On the Ke3 Monte Carlo only 1% of the Ke3 events were lost. We estimate the two electron contamination to be $(2 \pm 1)\%$.

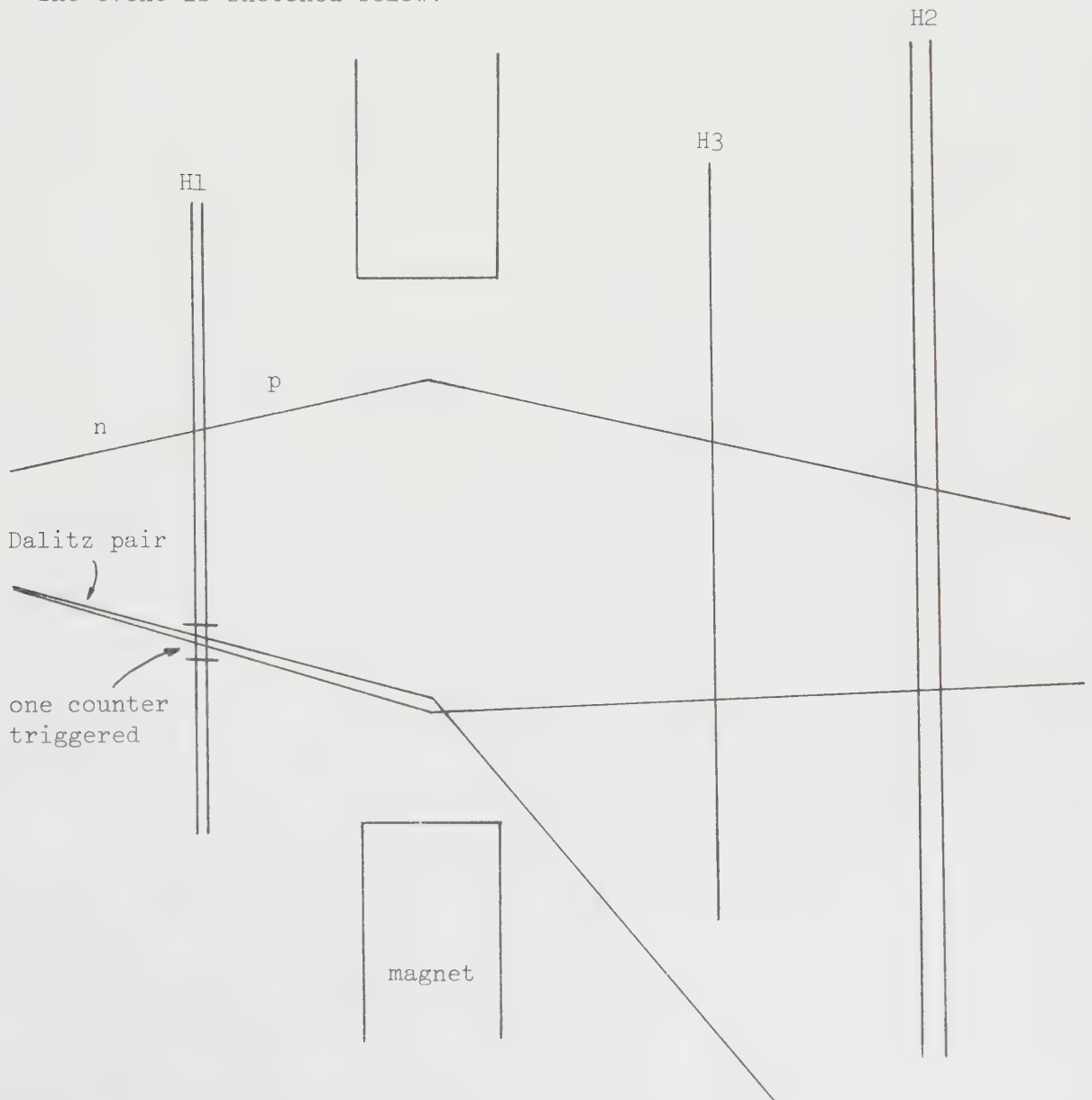
There remains the problem of those events with one electron. In principle this contamination is small; this was verified.

What kind of event can give this contamination? Consider the three pi decay of the K_L^0 ($K_L^0 \rightarrow \pi^+ \pi^- \pi^0$). In the first place, one of the charged pions must pass through the spectrometer; the other must not. Then, one of the gamma rays which comes from the pi zero must convert, and only one of the two electrons must pass through the spectrometer. We must remember that the detection of three particles

in any plane of any hodoscope constitutes a veto. In addition, the charge of the electron must be opposite to that of the pion which is detected. This mode of contamination is completely negligible.

There is another type of contamination which is perhaps not negligible - the accidental trigger in which a Dalitz pair passes through one scintillator in H1, coupled with a neutron interaction ($n p \rightarrow n p$) in another counter. This gives a two particle signal in H1. Should the magnetic field eliminate one of the electrons, the hodoscopes after the magnet see two particles, one an electron.

The event is sketched below:



Other two-particle, one-electron configurations can be fabricated. Most of these fail on the basis of a vertex cut or some other geometric consideration. To be absolutely certain no such event could systematically pass our filter, we rescanned one half of the final ensemble of events. This scanning was done by a physicist, using the listings from Artemis. Events were scanned and classified according to their "cleanness". Events with the possibility of a third track slightly off-time were noted. Events in which a freak Dalitz pair was in the least possible were also noted. Events reasonably supposed to be an interaction in H1 were classified as such. In short, half of the final sample was examined "under the magnifying glass" to be sure that there were no special freak contaminations of any importance. A sub-sample of absolutely pure events was compared with the entire sample. No significant differences in the distributions were detected. This examination was possible because the wire chambers have a reasonably long memory time; tracks which are not exactly in coincidence (and thus do not pass through triggered counters when reconstructed) also appear during the reconstruction. The efficiency of the chambers to oblique tracks allowed us to detect events such as the one sketched above.

We concluded that the contamination in the final sample from events other than Ke3 was of the order of a few percent. In the next section we examine the problem of improperly reconstructed events which, although true Ke3's, constitute a contamination.

IV. c. Preliminary Analysis and Cuts.

Two thousand, two hundred, twenty-five (2225) events remained after two-electron events of both types were removed. All these events were supposed to be Ke3 candidates. Below we describe a series of cuts, both geometric and kinematic, used to eliminate events which might bias our results. An event characterized by the Dalitz coordinates (E_e^*, E_π^*) was reconstructed at the point $(E_e^{*'}, E_\pi^{*'})$, due to various errors in the detection and reconstruction processes. In the Monte Carlo program we simulated these errors; nonetheless, events with large error, outside the bounds of that expected on the basis of the Monte Carlo program, had to be filtered out. A description of the kinematic reconstruction now follows.

Each event was geometrically reconstructed; the vector momentum of each particle and the disintegration vertex were calculated. The momentum was considered to be correct to within an $|\Delta p/p| = 3\%$. We quickly calculated the momenta using appropriate integrals determined from a detailed map of the magnetic field. The effect of the fringing field of the magnet was also taken into account when finding the vertex. A detailed description of the method used is found in the thesis of Suter.⁽¹³⁾ Using Monte Carlo events, the momentum analysis program was tested and found to be entirely satisfactory.

We formed the resultant vector, $\vec{P}_R$, the sum of the momenta of the two particles. We calculated the line of flight of the K_L^0 using the vertex and the target position. Then, the solution for the momentum of the disintegrating K_L^0 is given by:

$$\left| \vec{P}_K \right| = \frac{X \cdot P_{RL} \pm E_R \sqrt{X^2 - m_K^2 (P_{RT}^2 + m_R^2)}}{m_R^2 + P_{RT}^2}$$

where: E_R = energy of the resultant = $E_1 + E_2$
 m_R = mass of the resultant = $(E_R^2 - |\vec{P}_R|^2)^{\frac{1}{2}}$
 P_{RL} = component of the resultant parallel to
the K_L^O line of flight
 P_{RT} = component of the resultant perpendicular
to the K_L^O line of flight
 $X = (m_K^2 + m_R^2)/2$.

We called the two solutions for the K_L^O momentum PK1 and PK2. Each solution gives a different Lorentz transformation. We have a zero constraint fit; the neutrino goes backwards or forwards in the rest system of the K_L^O .

We now describe the cuts used to eliminate the following types of errors:

1. momentum or vertex reconstruction poor enough to shift an event unpredictably in the Dalitz plot;
2. reversed identification of the pion and the electron, changing the coordinates of the event completely;
3. change in position in the Dalitz plot due to the wrong choice of the K_L^O momentum in the lab (the two-fold kinematic ambiguity);
4. momentum or vertex reconstruction error such that conservation of energy-momentum is not satisfied.

The first series of cuts was made in the laboratory. The choice of the momentum solution in no way affects these geometric cuts, which follow:

- a cut on the particle momentum: events which are manifestly incorrectly reconstructed were rejected.
- a cut on the vertex position: events more than 3 meters away from the center of the magnet were prone to large measurement errors and were rejected.
- a cut on the entry angle of the K_L^0 : particles interacting in the collimator were rejected.
- a cut on the chi-squared of the adjusted tracks: scattered secondaries were rejected.
- a cut on the continuity of the tracks at the center of the magnet in both projections;
- a cut on the distance of closest approach of the two secondaries, the vertex being defined as the midpoint of this line; this eliminated accidentals which are not K_L^0 decays as well as poorly reconstructed $Ke3$ events.
- fiducial volume cuts eliminated events which might have touched a pole piece, etc.
- a cut on events at variance with the $2 \times 2 \times 2 \times 2$ trigger condition, as well as those events where the reconstructed tracks missed more than one of the triggered scintillators (within a margin for error).

We feel these cuts cover all the possible errors in reconstruction.

Under these conditions the resolution was a square of 20×20 MeV in

the (E_e^*, E_π^*) Dalitz plot. These cuts were responsible for the loss of about 600 events.

To protect against the possibility of inverse identification of the particles, we introduced the following cuts. We allowed two triggered groups in the Cerenkov counter, but eliminated all events where more than two groups triggered or where two groups were spatially non-contiguous (groups 1 and 3, for instance). We also eliminated events with a possible error in the electron identification, the path of the electron and the Cerenkov group triggered being at odds. These cuts eliminated another 200 events.

We have investigated the effects of the double solution for the K_L^0 momentum. We divide the events into three classes:

1. Either PK1 or PK2 is physically impossible (less than zero or greater than 2.5 GeV/c), the other solution being reasonable.
2. Both PK1 and PK2 are within physical limits, the difference between $(E_e^*, \text{solution 1})$ and $(E_e^*, \text{solution 2})$ being less than the resolution of 20 MeV. (Since the neutrino energy in the K_L^0 rest system does not depend on the choice of the solution, the pion energy difference is just the negative of the electron's.)
3. Both PK1 and PK2 are possible, the two solutions for the electron energy in the K_L^0 rest system being separated by more than 20 MeV.

We used the following procedure for treating the remaining events:

- Class 1 : choose the obvious solution;
- Class 2 : take the average between the two solutions;
- Class 3 : reject the event.

The inclusion of class 3 events changed the results. Since in their treatment an ambiguity exists, we rejected them. Several authors have treated this problem in another way. Each solution is treated as correct, giving two points on the Dalitz plot for each event. Each point is weighted according to the probability of its presence, based in part on the value of the K_L^0 momentum corresponding to the solution chosen. This method demands a detailed knowledge of the K_L^0 spectrum to carry out the weighting. We did not feel we knew our beam well enough to follow this procedure. The cut on the double solution eliminated 360 events.

As for kinematic cuts, we eliminated events where no solution in the K_L^0 rest system satisfied the conservation of energy-momentum. These events fall outside the limits of the Dalitz plot. We lost more events at the top and bottom than elsewhere. (Cutting the plot into horizontal strips, the top and bottom ones have the largest boundaries on the unphysical domain. Proportionally more poorly-reconstructed events are lost there.) Unfortunately, we know of no way of introducing a margin of error to save these events. As always we made identical cuts on the Monte Carlo events, we assume no large error.

In addition to the above, several cuts were applied to various kinematic variables in the K_L^0 rest system (the $|\vec{P}_K|$ having already been chosen). The loss from all kinematic cuts is 225 events.

The following table summarizes the events eliminated:

geometric cuts	600
doubtful electron identification	200
double solution	360
kinematic cuts	225

The final sample totalled 841 events.

V. Analysis.

The underlying principle of the analysis is not complicated. Suppose - The detector has an acceptance of 4π and a constant efficiency. There are no measurement errors. The difficulties introduced by the two-fold kinematical ambiguity are ignored. Then - An event with the coordinates (E_e^*, E_π^*) will be reconstructed at the point (E_e^*, E_π^*) . The distribution of these events will represent the true distribution. The distribution of these events fitted to the functional form of the theoretical probability distribution will yield values for the free parameters involved.

Reality was more complex. The observed distribution was not the true one, but one modified by the limited acceptance of the spectrometer. This acceptance, not uniform in the lab, resulted from purely geometric considerations as well as various detection efficiencies. Finally, because of measurement errors explained in chapter 4, an event characterized by the point (E_e^*, E_π^*) was reconstructed at the coordinates $(E_e^{*'}, E_\pi^{*'})$. Events can migrate. To determine the value of the significant parameter, λ_+ , the experimental distribution had to be compared with the Monte Carlo prediction, where the simulation takes into account the factors mentioned above as well as the functional dependence of the physics.

V. a. Monte Carlo Program.

A general Monte Carlo program was written to simulate the

experiment. The program generated events uniformly in phase space and using a beam momentum distribution approximating the predicted beam conditions transformed them into the laboratory system. The events were weighted to correct for the vector interaction (with f_+ constant) and the adjusted beam profile. We discuss here this part of the Monte Carlo; in the next section we discuss the further weightings necessary to account for the various detection efficiencies.

The primary generation of the events was done according to phase space. The kinematic variables were chosen to satisfy energy-momentum conservation in the rest system of the K_L^0 . This fixed the relative directions of the three secondaries. The triad was rotated randomly to ensure isotropy in the K_L^0 rest system. The K_L^0 direction in the lab was next chosen, and the point of disintegration along this direction was randomly chosen according to the usual exponential decay law. The triad of secondaries was then Lorentz-transformed to the lab system, and the two charged secondaries tracked through the spectrometer. Events failing to pass through the spectrometer apertures, as well as those failing to satisfy the trigger conditions were dropped from further consideration.

At this point we recorded on tape all events accepted by the spectrometer. We then modified their distribution by weighting them to take into account other factors. (The weights could be normalized to yield a sample of arbitrary size.)

For instance, we have seen in chapter 2 the probability distribution for $Ke3$ with a pure vector interaction is :

$$\frac{d^2W}{dE_e^* dE_\pi^*} = f_+^2(q^2) \overbrace{\left[(E_\pi^* - A) + 2E_e^* E_\nu^* \right]}^{P_\nu},$$

where A is the maximum energy of the pion. We thus attribute to each event a weight P_ν to account for this difference from phase space. Figure 9 shows the departure from phase space given by the vector interaction. The actual distribution differs from figure 9 because of the spectrometer acceptance.

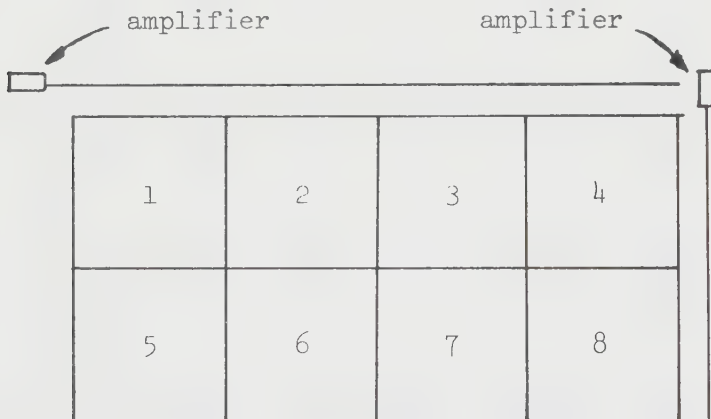
The K_L^0 momentum spectrum in the Monte Carlo was adjusted to agree with the resultant momentum of the two charged particles in the decay. This corresponds to attributing to each event a weight P_B , the ratio of the number of events of its momentum in the actual beam profile to the number of events of that momentum in the simulated beam. The value of λ_+ is insensitive to reasonable changes in this weighting.

V. b. Detection Efficiencies.

We now describe the factors introduced into the Monte Carlo program to simulate detection efficiencies and measurement errors affecting the results.

Wire chamber efficiencies - For this study, we selected a sample of two track events and we divided each gap into eight zones. (Choosing events of two tracks eliminated those resulting from false triggers.) Treating each track individually, we took each gap in turn, "erasing" all the sparks in the gap. Then, we tried to reconstruct the track in the absence of the aforementioned gap. If the track

could not be reconstructed, it was dropped. If the track was reconstructed, we searched in the "muted" gap to see if a spark was present. The efficiency of each zone in each gap was defined as the number of times a spark was present near the reconstructed track divided by the total number of reconstructed tracks. Since the horizontal and vertical projection were independently recorded, three efficiencies for each zone could be calculated: the readout efficiency for the horizontal projection, the readout efficiency for the vertical projection, and the local sparking efficiency of the gap. The probability of detection for a given projection is the readout efficiency times the sparking efficiency. The following sketch shows the division of a gap into zones, the numbering refers to the table of efficiencies found on the next page.



The beam direction is out of the page.

Wire Chamber Efficiencies

		zone							
		1	2	3	4	5	6	7	8
gap									
1		175	933	976	305	133	769	605	197
		.70	.74	.75	.72	.66	.67	.67	.61
		.52	.62	.70	.74	.41	.51	.62	.68
2		249	1005	1047	431	176	761	604	254
		.48	.61	.60	.44	.35	.56	.54	.37
		.34	.53	.59	.53	.28	.47	.53	.49
3		28	953	1117	120	15	699	712	36
		.78	.86	.82	.76	.86	.85	.79	.77
		.53	.73	.80	.69	.66	.72	.77	.74
4		58	1007	1125	138	27	670	657	60
		.55	.76	.78	.76	.66	.83	.81	.78
		.44	.59	.73	.76	.55	.71	.78	.73
5		135	700	767	318	77	553	482	222
		.62	.77	.81	.72	.63	.69	.77	.70
		.56	.66	.69	.63	.33	.56	.63	.57
6		205	949	873	364	132	813	590	242
		.57	.73	.71	.64	.43	.66	.70	.51
		.27	.29	.51	.60	.08	.20	.40	.52
9		121	740	976	297	30	596	663	186
		.78	.74	.70	.56	.69	.73	.64	.53
		.72	.70	.74	.61	.59	.60	.68	.57
10		117	961	1017	297	40	689	629	159
		.56	.58	.70	.60	.57	.57	.65	.64
		.43	.52	.63	.59	.47	.50	.65	.65

The first line for each gap is the number of reconstructed tracks;
the second is the horizontal efficiency, the third line the vertical efficiency.

Except for certain zones with too few reconstructed tracks, we feel two-figure precision justified. The method did not take into account possible correlations between efficiencies in different gaps. For example, if there existed an inefficiency in all the chambers along a certain line of flight, the efficiencies are too high.

The efficiency calculation also allowed examination of the residues in the gaps. A residue is the distance between the spark found in a muted gap and the point of passage of the track reconstructed without that spark. The residue distribution showed the measurement error introduced by the magnetostrictive readout system and the effects of the multiple scattering.

We simulated the effects of the errors in reconstruction in the following manner. Each track was traced through the spark chambers. At each gap, we decided whether the spark would be detected, on horizontal and on vertical, by using a random number generator and the zonal detection efficiency of the gap. If the spark was kept, it was randomly displaced from its original position in accordance with the residues previously calculated. At the end of the procedure for both tracks, the event was examined to see if enough sparks were left for reconstruction. If not, the event was rejected. About 65% of the Monte Carlo events were lost in this way. This is in good agreement with the loss of real events from the same cause.

The events which survive this process were then reconstructed using the same program as the experimental events. There were errors

in the momenta and angles due to spark position displacements as well as reconstruction errors caused by a marginal number of sparks.

The above method for spark chamber efficiencies does not depend on any other experimental inefficiencies. Suppose the left half of the spectrometer was much less efficient than the right half. This would influence the total number of reconstructed tracks in each half, but not the calculation of the wire chamber efficiencies.

Cerenkov counter efficiency - Since hodoscope inefficiencies can simulate Cerenkov counter inefficiencies, finding the local efficiency of the Cerenkov is somewhat more subtle than the problem above. To perform an independent calibration we first scanned a sample of films looking for distinct showers in the optical chambers. We were extremely selective in this search. The shower had to be unmistakable, produced by an electron and not by a converted gamma ray. No other track could be present which might have triggered the counter; a second or third track could not be even a doubtful electron. We then tried to reconstruct the track from the wire chamber information, obtaining its position and orientation in space. We calibrated the local efficiency by examining whether or not the Cerenkov counter was triggered. Since any event with one well-identified electron was usable, we used rejected Ke3 events where the pion track had proved unreconstructable, as well as experimental events from outside the Ke3 sub-sample. Since the calibration events were not identical to the Ke3 events, we were somewhat relieved of the problem of the correlation of efficiencies with other elements.

We had a sample of 300 well-identified electrons for the

calibration. Because of the distribution of the showers, this was adequate for the center but marginal for the sides of the counter. The problem of few showers near the sides was compounded by the low efficiency of the counter there. To have a high statistics situation in these regions would have demanded a huge scanning effort. We note that all 300 showers were verified by a physicist.

We divided the Cerenkov counter into several different configurations to estimate its local efficiency. One method looked at the up-down asymmetry of the counter. Another calculated the efficiency of each group of photomultipliers. This was complicated because a good fraction of the tracks triggered two groups of photomultipliers, and many tracks which did not trigger the Cerenkov counter at all could have. The simplest method was to divide a plane parallel to the entrance window into zones and to calculate the efficiency averaged over each zone. This was done for the plane at center of the magnet and in another test at H2, just behind the exit of the counter. The results were similar.

In the Monte Carlo, each event was further weighted by P_{CER} , the probability of its being detected based on the electron trajectory.

H3 efficiency - There remained one more weighting to be introduced in the Monte Carlo. The hodoscope H3 gave a different distribution of triggered scintillators than predicted by Monte Carlo. We were unable to reproduce this distribution by any of the other calculated efficiencies. One of the large scintillators (H3-1) appeared to be very inefficient; it is possible that its photomultiplier was affected by the stray magnetic field. Although symmetry checks of the

efficiencies were made on-line, such checks were based on all the triggers, whereas the Ke3 events are but a small subsample of the total. In any event, we were forced to introduce another weight, P_{H3} , the efficiency of this hodoscope.

In summary, the i^{th} event was characterized by a weight P_i , given by:

$$P_i = P_{vi} \cdot P_{Bi} \cdot P_{CERi} \cdot P_{H3i} .$$

Figures 10 and 11 show the agreement between the Monte Carlo weighted in this way and the Ke3 subsample. The number of Monte Carlo events was normalized to equal the number of experimental events.

V. c. Dalitz Plot Analysis.

At this point we have both a sample of experimental events, the i^{th} event being characterized by the coordinates $(E_{ei}^*, E_{\pi i}^*)$ and a sample of Monte Carlo events, each characterized by a pair of coordinates in the same space, with a weight P_i expressing the probability that the event was detected. The total number of theoretical events is given by:

$$N^{th} = \sum_i f_+^2(q_i^2) \cdot P_i ,$$

or, parametrizing $f_+(q^2)$ in the usual way:

$$f_+(q^2) = f_+(0) \cdot (1 + \lambda_+ q^2/m_\pi^2) ,$$

$$N^{th} = f_+^2(0) \sum_i (1 + \lambda_+ q_i^2/m_\pi^2)^2 \cdot P_i .$$

We normalize the total number of events by multiplying the weights by an appropriate factor and grouping all the normalization constants into one constant C where:

$$C = N^{exp}/N^{th} ,$$

yielding

$$N^{th} = C \sum_i (1 + \lambda_+ q_i^2/m_\pi^2)^2 \cdot P_i .$$

The total number of theoretical events equals the total number of experimental events.

We divided the Dalitz plot into squares of 20 MeV by 20 MeV. We regrouped squares with fewer than ten experimental events. The analysis consisted of varying the parameter λ_+ to minimize the following quantity:

$$\chi^2 = \sum_j \frac{(N_j^{th} - N_j^{exp})^2}{N_j^{th}}$$

where j refers to the j^{th} cell of the Dalitz plot.

To calculate N_j^{th} as a function of λ_+ , we write

$$N_j^{th} = \sum_{i \in j} \left[1 + 2\lambda_+ q_i^2/m_\pi^2 + \lambda_+^2 (q_i^2/m_\pi^2)^2 \right] \cdot P_i ,$$

where the summation is over all i in the j^{th} cell. Then:

$$N_j^{th} = \sum_{i \in j} P_i + \lambda_+ \sum_{i \in j} 2P_i q_i^2/m_\pi^2 + \lambda_+^2 \sum_{i \in j} P_i (q_i^2/m_\pi^2)^2$$

Noting the sums as follows:

$$W1_j = \sum_{i \in j} P_i , \quad W2_j = \sum_{i \in j} 2P_i q_i^2/m_\pi^2 , \quad W3_j = \sum_{i \in j} P_i (q_i^2/m_\pi^2)^2 ,$$

we write the number of theoretical events in the cell j as:

$$N_j^{th} = W1_j + W2_j \cdot \lambda_+ + W3_j \cdot \lambda_+^2 .$$

The computational advantage of this method is obvious: for each sample of events, we calculated the set of weights $W1$, $W2$, $W3$ once and once only. Then by varying the parameter λ_+ we could change the population of the j^{th} cell of the Dalitz plot. The value of the parameter λ_+ which minimized the above chi-squared was taken as the correct one. Each time we influenced the population of the Dalitz plot (by changing the cuts, for example) we were obliged to recalculate the weights $W1$, $W2$, $W3$.

We note that an event found in the j^{th} cell may have been created in the j'^{th} cell and then migrated during the reconstruction process. Its weight due to the vector interaction is nonetheless the weight it

had at its point of creation, as explained at the beginning of this chapter.

In order to minimize the chi-squared as a function of λ_+ we used the program Minuit.⁽¹⁴⁾ The program uses an iterative method for finding the minimum and furnishes the standard deviation error on the minimum. It also gives the minimum chi-squared as a test of the validity of the result.

This method of analysis has the advantage of fitting both the electron and pion spectra simultaneously, due to its two-dimensional nature. It has the disadvantage of supposing an infinitely large Monte Carlo sample; since the Monte Carlo sample is normalized to the experimental sample, the statistical error introduced by the finite size of the Monte Carlo is ignored.

Since q^2 is a function of E_π^* only, there is another method of performing the analysis.

V. d. Analysis by Strips.

We divided the Dalitz plot into horizontal strips of 20 MeV width. We calculated, after normalization, the number of Monte Carlo events in each strip with λ_+ equal to zero. Plotting $(N_k^{\text{exp}}/N_k^{\text{th}(\lambda_+=0)})^{1/2}$ as a function of $\langle q_k^2 \rangle / m_\pi^2$, where k refers to the k^{th} strip, the slope of the resulting straight line is λ_+ . This follows from

$$\frac{N_k^{\text{exp}}}{N_k^{\text{th}(\lambda_+=0)}} = \frac{f_+^2(0) (1 + \lambda_+ \langle q_k^2 \rangle / m_\pi^2)^2}{f_+^2(0)},$$

and taking the square root of both sides

$$\sqrt{\frac{N_k^{\text{exp}}}{N_k^{\text{th}(\lambda_+ = 0)}}} = 1 + \lambda_+ \langle q_k^2 \rangle / m_\pi^2 .$$

There remained two problems: how to calculate the error on the y-coordinate and how to calculate the x-coordinate itself, that is, how to calculate the average value of q^2 in each strip.

Let n_e be the number of experimental events in a given strip, n_t the number of theoretical events, and n_t' the number of non-normalized theoretical events. We know that the sum of all the n_e is equal to the sum of all the n_t by the normalization condition. Let R equal the ratio of the total number of non-normalized theoretical events to the total number of experimental events. We calculated the error in the quantity Y , where

$$Y = \sqrt{\frac{n_e}{n_t}} = \sqrt{\frac{R n_e}{n_t'}} .$$

Using

$$\sigma_Y^2 = \left(\frac{\partial Y}{\partial n_e} \right)^2 \sigma_{n_e}^2 + \left(\frac{\partial Y}{\partial n_t'} \right)^2 \sigma_{n_t'}^2$$

we found

$$\sigma_Y^2 = \frac{1}{4 n_t^2} \left[n_t + n_e (1/R) \right] .$$

When we did the fit to the straight line, we attributed to each point

a weight inversely proportional to its σ^2 . For the i^{th} point the weight is given by:

$$w_i = 4n_{ti}^2 / (n_{ti} + n_{ei}(1/R)) .$$

For an infinitely large Monte Carlo sample, $R \rightarrow \infty$ and $\sigma_Y = 1/(2 \sqrt{n_t})$.

The second problem concerned the value of $\langle q^2 \rangle$ for the x-coordinate of the points. There were three choices: (1) the geographic center of the strip; (2) the average taken over the experimental events in the strip; (3) the average taken over the Monte Carlo events in the strip, using the value of q^2 of each event at its point of generation. The third choice is consistent with the rest of the experiment; it assigns values at the point of generation of the event, regardless of the point of reconstruction. This was the procedure used to assign the weights for the vector interaction. Bloch⁽¹⁵⁾ has shown that in fact the third choice is correct. In physical terms using the experimental events to find the average q^2 introduces the same error twice: a shift in the position of the event distorts the population and in addition it falsifies the average q^2 . The Monte Carlo events also distort the distribution, but the q^2 average is unchanged because the values of q^2 are taken at the point of generation and are thus correct.

We calculated our slopes on the basis of procedures 2 and 3 and observed varying size differences between the two methods of averaging depending on the sample of events selected. The results given in the next chapter using the strip method are all based on the third method of averaging.

Finally, we note that the strip method ignores the goodness of fit of the electron spectrum in each strip.

VI. Results and Conclusions.

We analyzed our sample of 841 events using the methods outlined.

The results are summarized in the following table.

Sample	Number of Events	λ_+ Dalitz plot	λ_+ strip method
all events	841	0.038 ± 0.014 $\chi^2/\text{NDF} = 3.5$	0.039 ± 0.015 $\chi^2/\text{NDF} = 1.1$
all events (different weighting)	841	0.043 ± 0.014 $\chi^2/\text{NDF} = 3.8$	0.044 ± 0.016 $\chi^2/\text{NDF} = 1.3$
pi minus, electron plus	338		0.049 ± 0.020 $\chi^2/\text{NDF} = 0.88$
electron minus, pi plus	503		0.044 ± 0.014 $\chi^2/\text{NDF} = 0.53$

The first result is the entire ensemble of events with the weights calculated as explained in chapter 5. A map of these weights is given in figure 12. To be certain our results did not crucially depend on this weighting, we recalculated the value of lambda plus using the same sample, but with a different set of weights. The essential differences lay in the beam profile and the H3 efficiency. The fits to the

distributions in the lab did not change appreciably. As a further check, we divided the sample into two subsamples, representing the two charge states. The value of lambda plus is compatible between the two subsamples. We feel there was no bias which depended on the sign of the electron.

Figure 13 shows the Dalitz plot for the first result. The first line in each cell represents the number of experimental events. The second and third lines show, respectively, the number of Monte Carlo events with λ_+ equal to zero and with λ_+ equal to the final adjusted value. The linear fit by strips is shown in figure 14.

The elevated value of the chi-squared in the Dalitz plot was due to a few squares (or rectangles) where the difference, even after the fit, is large. We eliminated these squares and found the value of lambda plus unchanged. Essentially this was a manifestation of a bad fit to the electron spectrum in one or two strips. We were not able to alter this by varying any of the cuts.

The errors indicated in the table are purely statistical, and are on the order of 0.014. By varying the cuts for a number of different samples, we estimated our systematic error to be 0.008. Adding these errors in quadrature, our value for lambda plus is:

$$\lambda_+ = 0.041 \pm 0.016 .$$

This value can be compared to the following results for lambda plus in $K^0 e3$ decay:

<u>Authors</u>	<u>Year</u>	<u>Number of Events</u>	<u>Value</u>
Aronson ⁽¹⁶⁾	1968	1000	0.02 ± 0.013
Basile ⁽⁵⁾	1968	7233	0.023 ± 0.012
Bisi ⁽¹⁾	1971	42000	0.023 ± 0.005
Chien ⁽²⁾	1972	16000	0.05 ± 0.01
Neuhofer ⁽¹⁷⁾	1972	1910	0.022 ± 0.014
Albrow ⁽²⁰⁾	1971	-	0.055 ± 0.011

We have indicated only the most recent results. As a matter of record, the world average given by Chounet⁽¹⁸⁾ in 1970 was 0.017 ± 0.007 . The last two years have seen two high statistics experiments, those of Bisi et al. and Chien et al. The experiment of Bisi is in excellent agreement with that of Basile et al., and in agreement with the prediction of $K^*(890)$ dominance; the experiment of Chien is in disagreement with these results.

The experiment of Neuhofer et al. is especially interesting; their 1910 events were completely kinematically determined. The K^0 came from the reaction $K^+ p \rightarrow K^0 p \pi^+$. The K^0 momentum was known, the problem of kinematic ambiguity eliminated. Using all their events, they found a lambda plus 0.013 higher than their final result. The lower value - the final one quoted by the authors - came from the elimination of events with large measurement errors on the pion energy; this preferentially cuts the low energy pions, which have a larger relative error than the rest. (The authors could determine the resolution of the pion energy in the rest system of the K^0 without ambiguity.) They note that perhaps this kinematic singularity in the pion

energy resolution explains the high values for λ_+ in the experiments of Chien and Albrow et al. Their reasoning is the following: the spectrometers of Chien and Albrow were quite sensitive in the region of high q^2 (low E_π^*); this sensitivity in a region with a singularity in the resolution could account for abnormally high values of λ_+ . Conversely, Bisi had a spectrometer acceptance going rapidly to zero at high q^2 , thus being unaffected by such a singularity. Our acceptance is shown in figure 15. We were very sensitive in the high q^2 region; whereas our result is higher than that of Bisi, we are not quite so high as Chien and Albrow. We feel that this is an open question and should be seriously investigated in future experiments.

We investigated the possibility of a quadratic term in the form factor expansion:

$$f_+(q^2) = f_+(0) \left[1 + \lambda_+ q^2/m_\pi^2 + \lambda_+' (q^2/m_\pi^2)^2 \right].$$

We find our data incompatible with a positive λ_+' , in disagreement with the compilation of Gaillard et al.⁽¹⁹⁾ To insist on a quadratic term, we would need an abnormally high value for λ_+ and a large negative value for λ_+' . We excluded this possibility and concluded that within the limits of our statistics there is no quadratic term in q^2 .

It is interesting to compare the value for λ_+ in $K^0 e3$ decay to that in $K^+ e3$ decay. If the $|\Delta I| = \frac{1}{2}$ rule is good, we expect the values to be the same. Winter⁽²⁰⁾ gives the following world average, admitting that he sometimes averaged incompatible results:

$$\lambda_+ (K^0 e3) = 0.031 \pm 0.004 ,$$

$$\lambda_+ (K^+ e3) = 0.029 \pm 0.006 .$$

The agreement is good.

The value of λ_+ for $K^0 e3$ decay should be the same as that for $K^0 \mu3$ decay, because of $e-\mu$ universality. The agreement here is poor, as given by Winter:

$$\lambda_+ (K^0 e3) = 0.031 \pm 0.004 ,$$

$$\lambda_+ (K^0 \mu3) = 0.079 \pm 0.008 .$$

Probably, the problem here is in the $K\mu3$ experiments which are much more difficult and prone to systematic errors.

Our results are compatible with the world average for $K^0 e3$ given by Winter. Our error bars are too big to distinguish between the two poles, around 0.025 and around 0.050. However we exclude a large value for λ_+ , say 0.08 or 0.09. We are about one standard deviation from the value predicted by $K^*(890)$ dominance. We find no evidence for a quadratic term.

That two very high statistics experiments are in disagreement suggests that the important errors in $K^0 e3$ are of a systematic nature. While the spark chamber technique allows the experimenter to gain a factor of about ten in the number of events over bubble chamber techniques (because a non-uniform acceptance can be used to favor the bottom of the Dalitz plot), the inherent uncertainty in the correctness

of Monte Carlo calculations must always be a source of worry. The ultimate resolution of the problem lies not in extremely high statistics experiments, but in those where the sources of systematic bias from the Monte Carlo program are minimized.

References

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(This is the most recent in a long series of articles published by this group; first preliminary results were given as early as the Kiev conference in 1970, where a value of λ_+ in Ke3 of 0.08 ± 0.01 was presented.)
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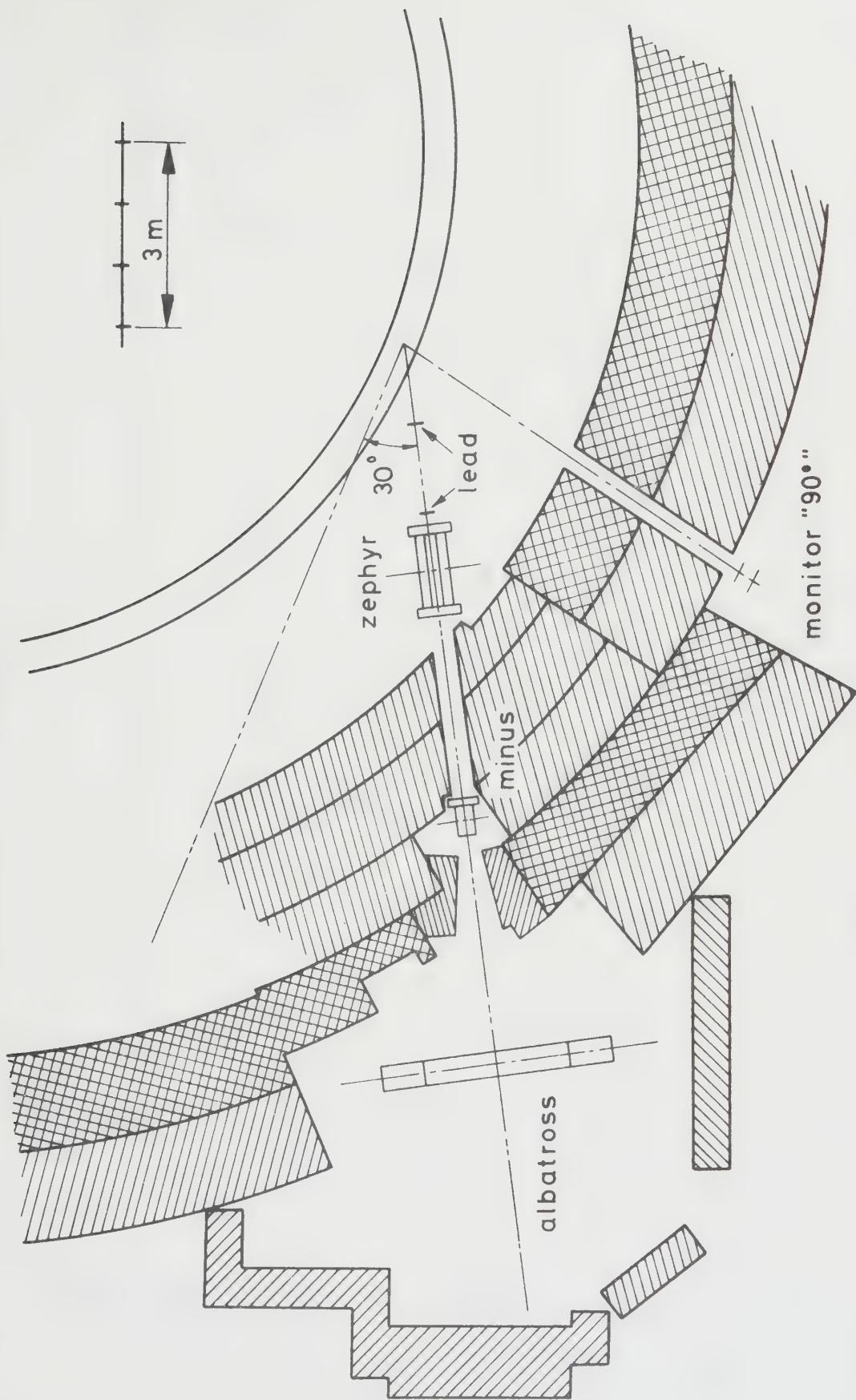


FIGURE 1 EXPERIMENTAL ZONE

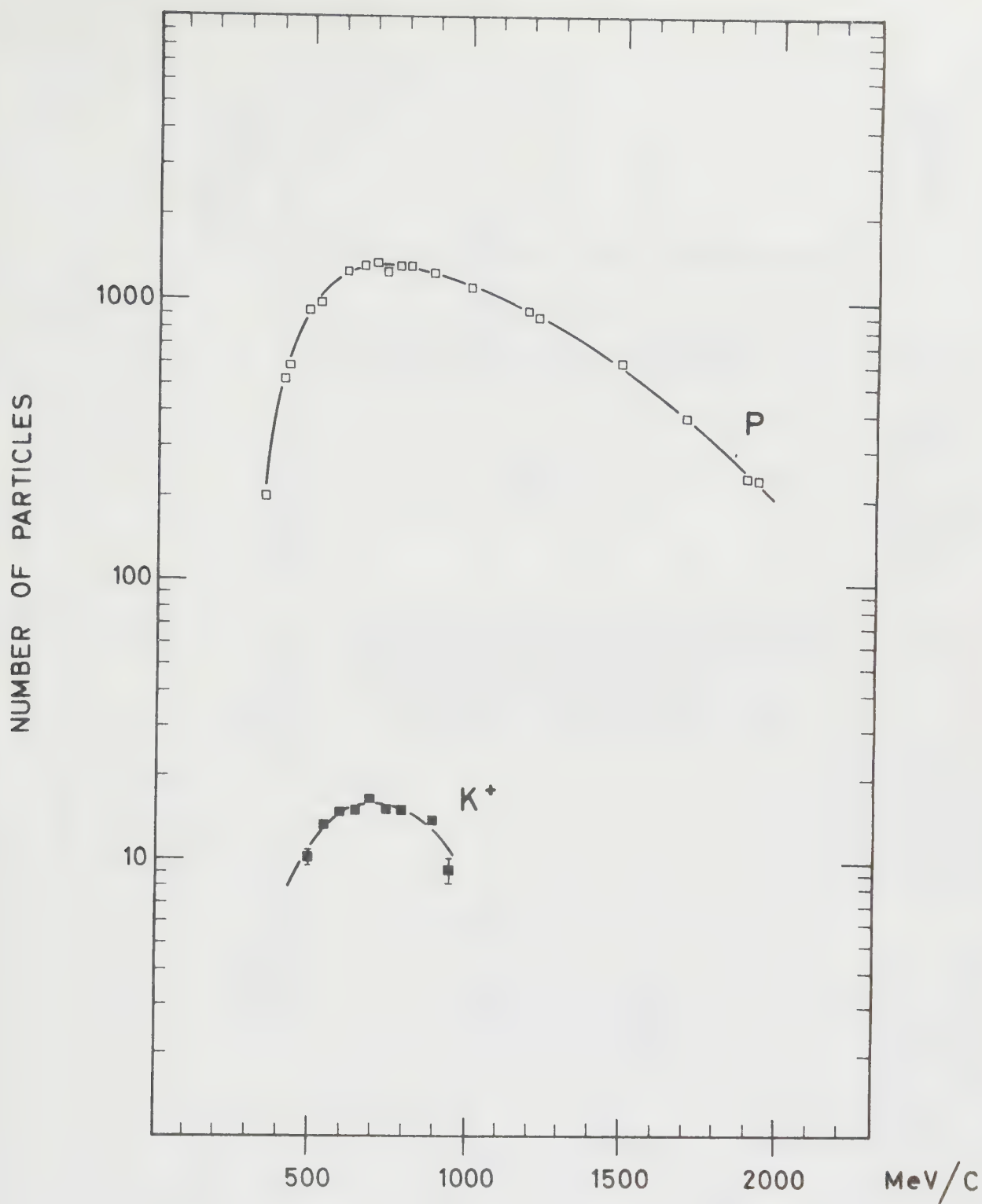


FIGURE 2

RELATIVE PRODUCTION
CROSS-SECTIONS BY
3 GEV PROTONS
AT 30°

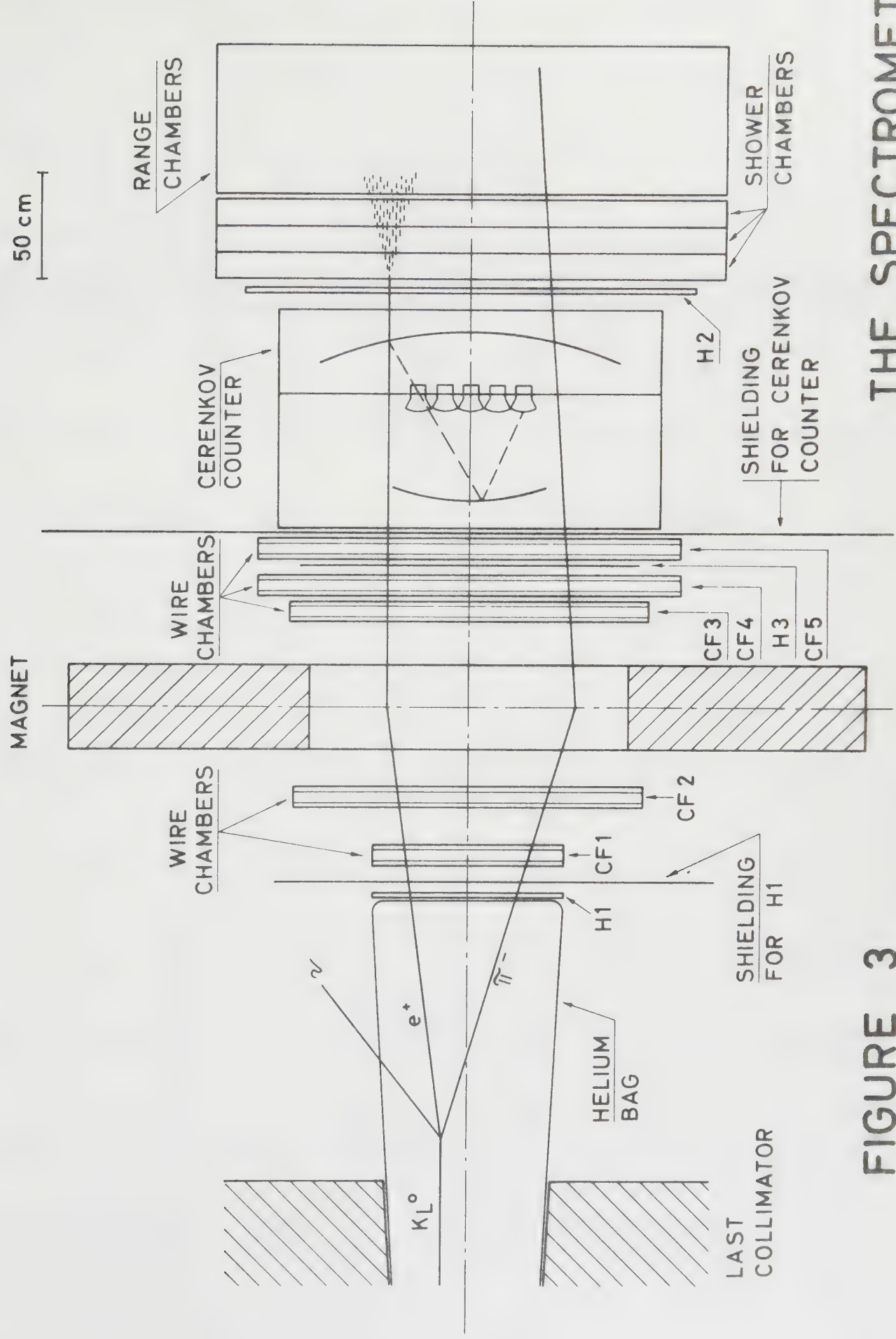


FIGURE 3

10 cm

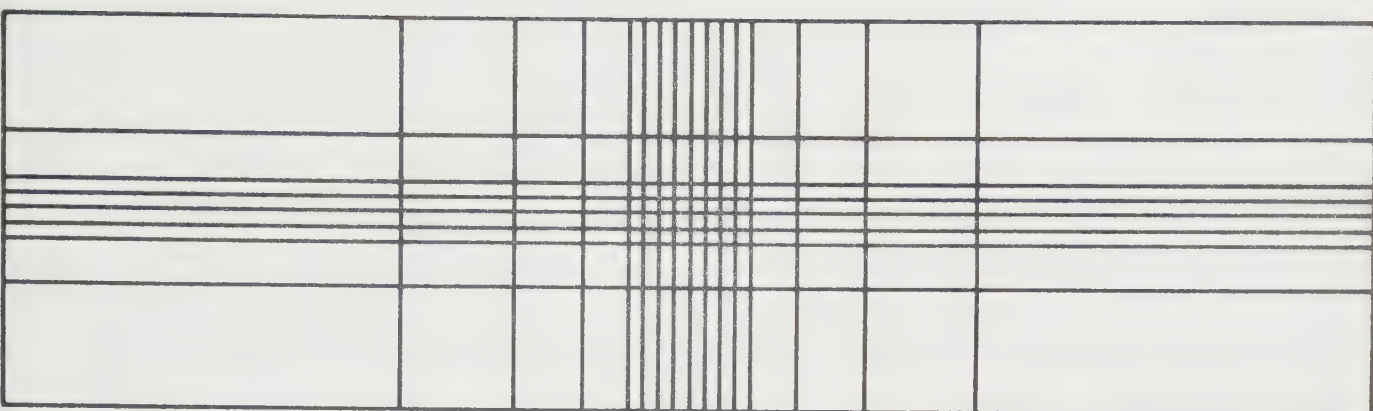


FIGURE 4a

HODOSCOPE
H1

20 cm

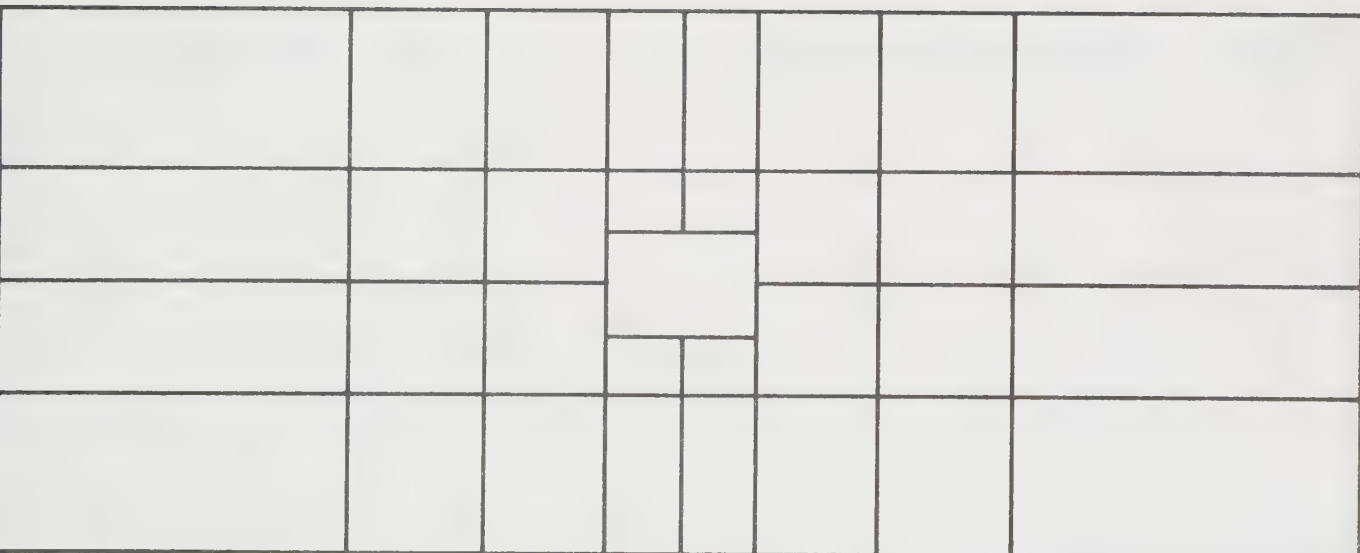


FIGURE 4b

HODOSCOPE
H2

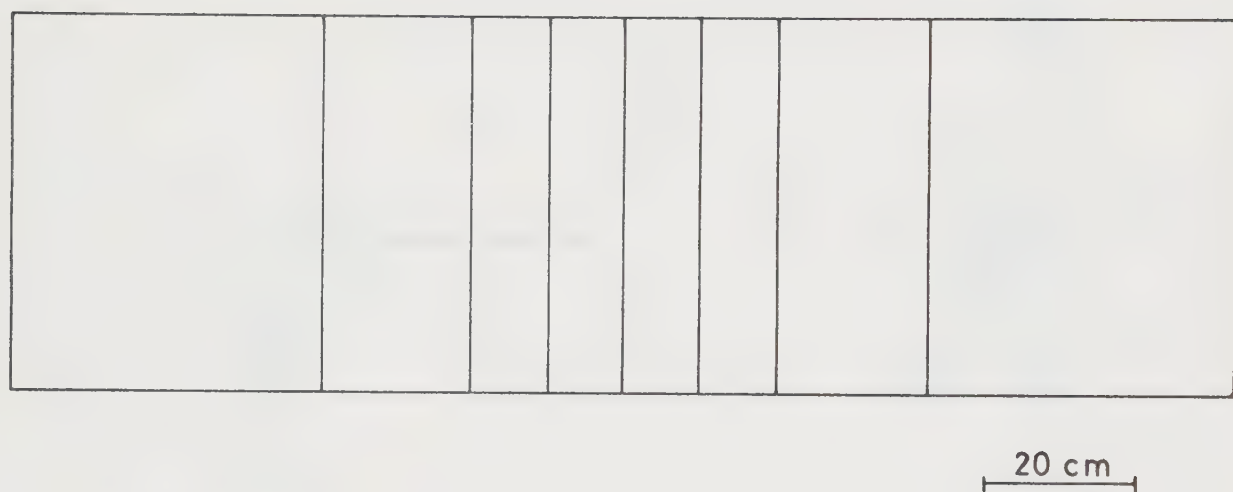


FIGURE 5

HODOSCOPE H3

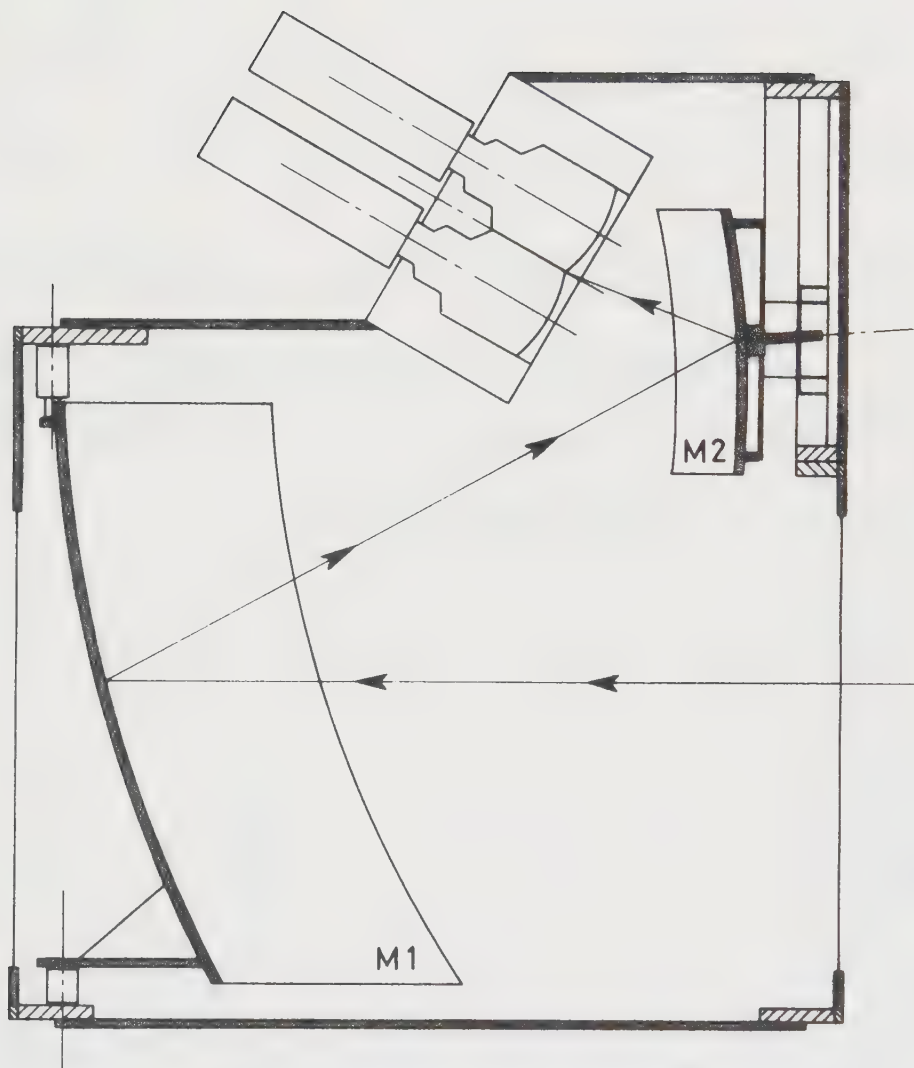
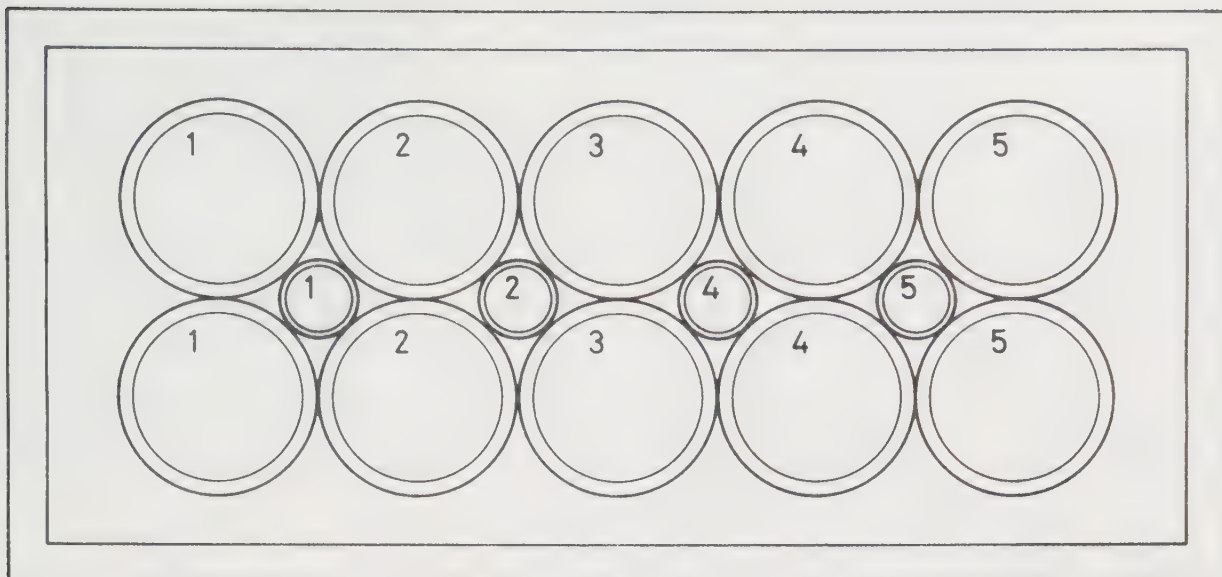
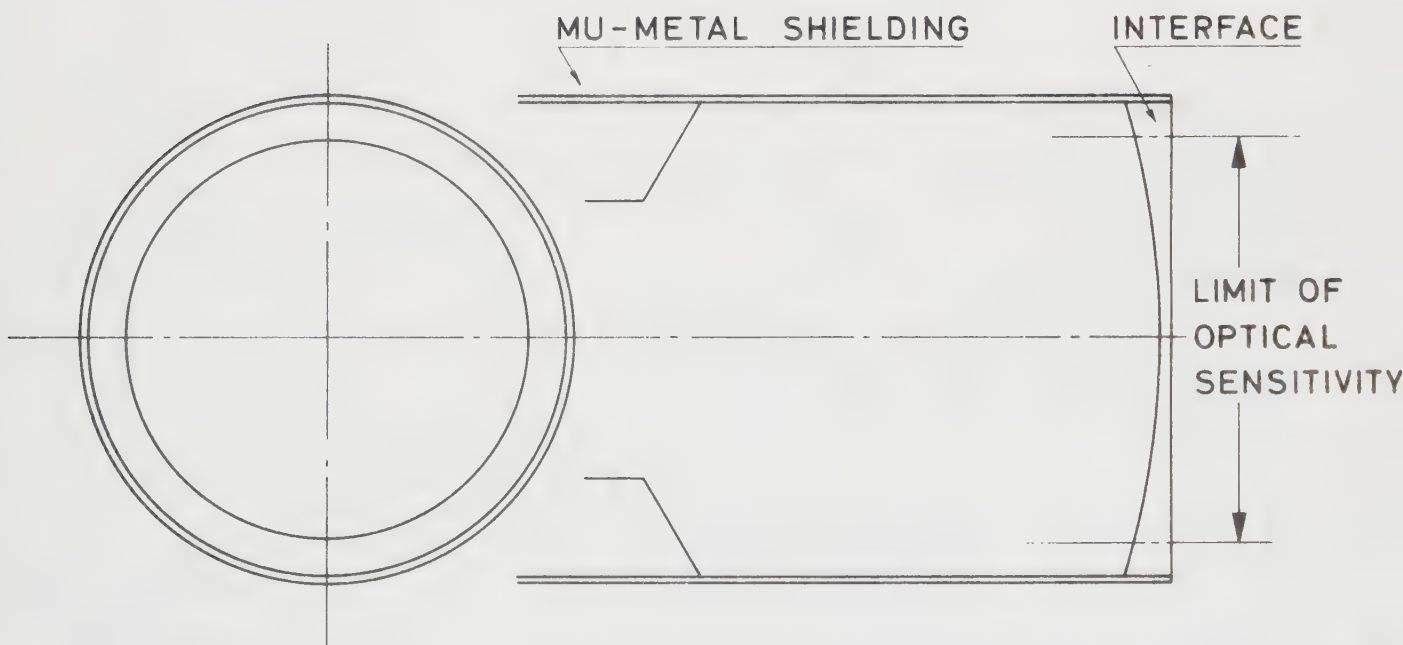


FIGURE 6 CERENKOV COUNTER



ARRANGEMENT OF PHOTOMULTIPLIERS IN THE CERENKOV COUNTER

(NUMBERS REFER TO ELECTRONIC GROUPING)



DETAIL OF A 58 DVP
PHOTOMULTIPLIER

FIGURE 7

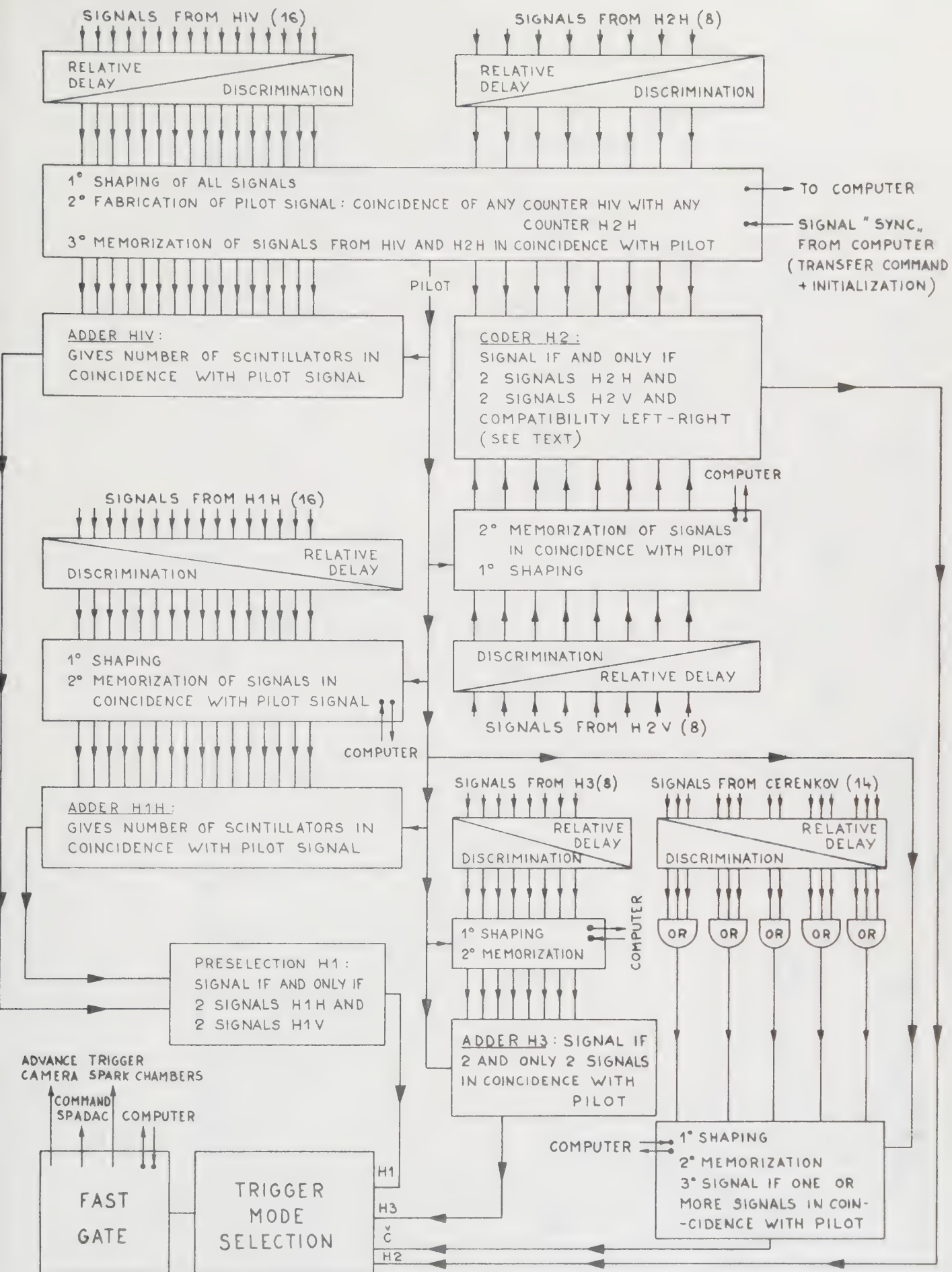


FIGURE 8

Ke3

FAST LOGIC

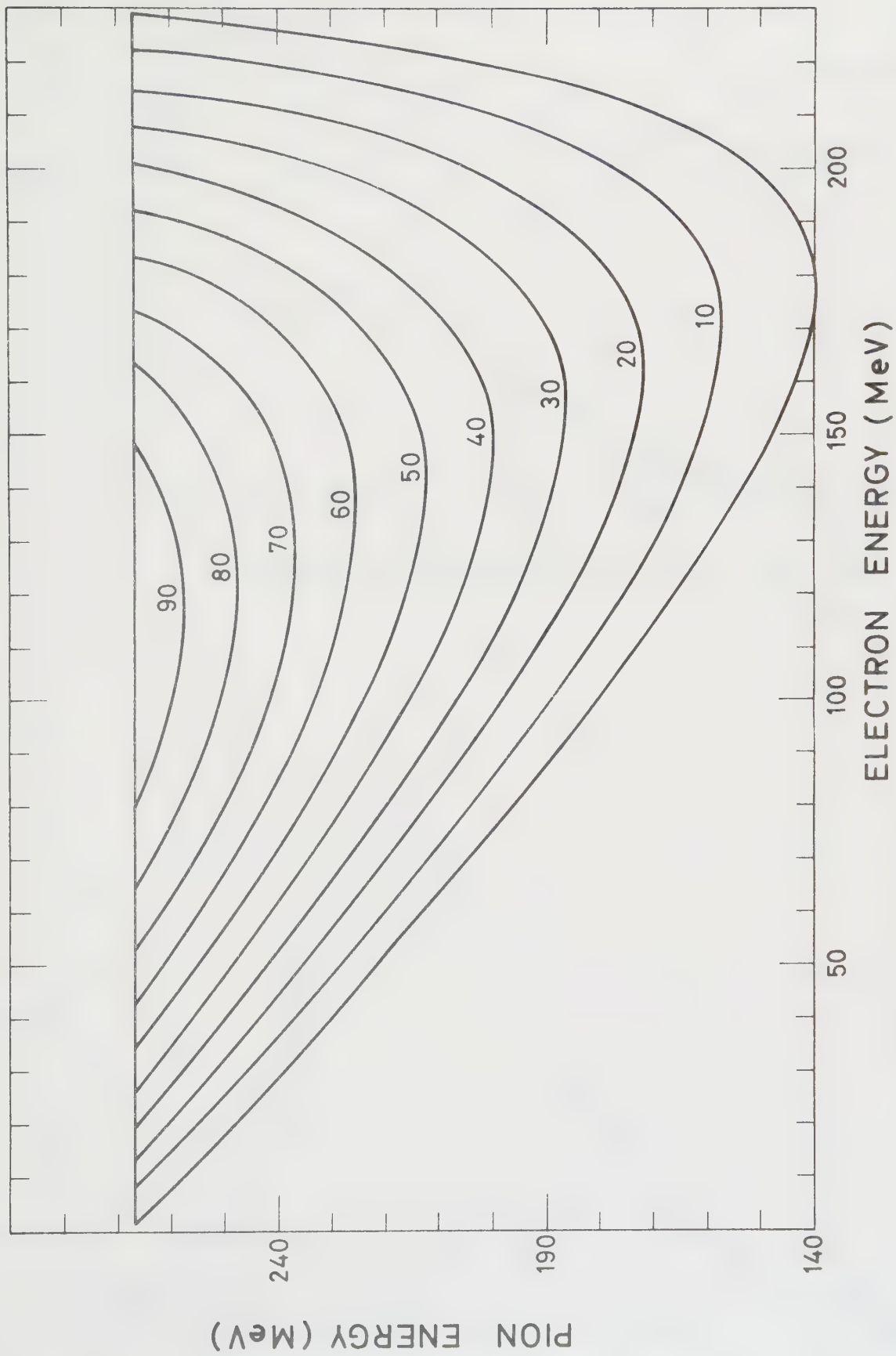


FIGURE 9 CONTOUR PLOT FOR THE
VECTOR INTERACTION

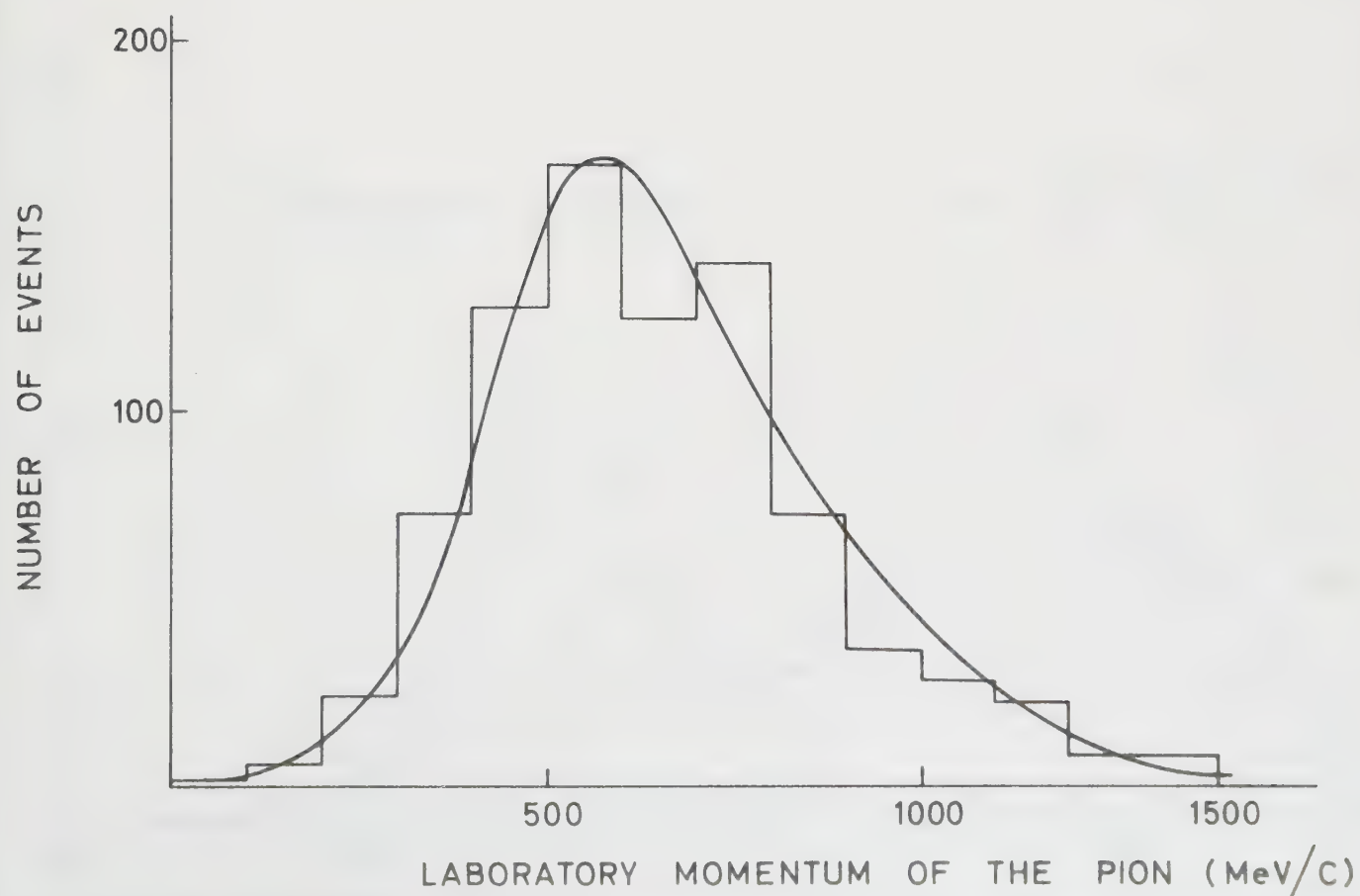
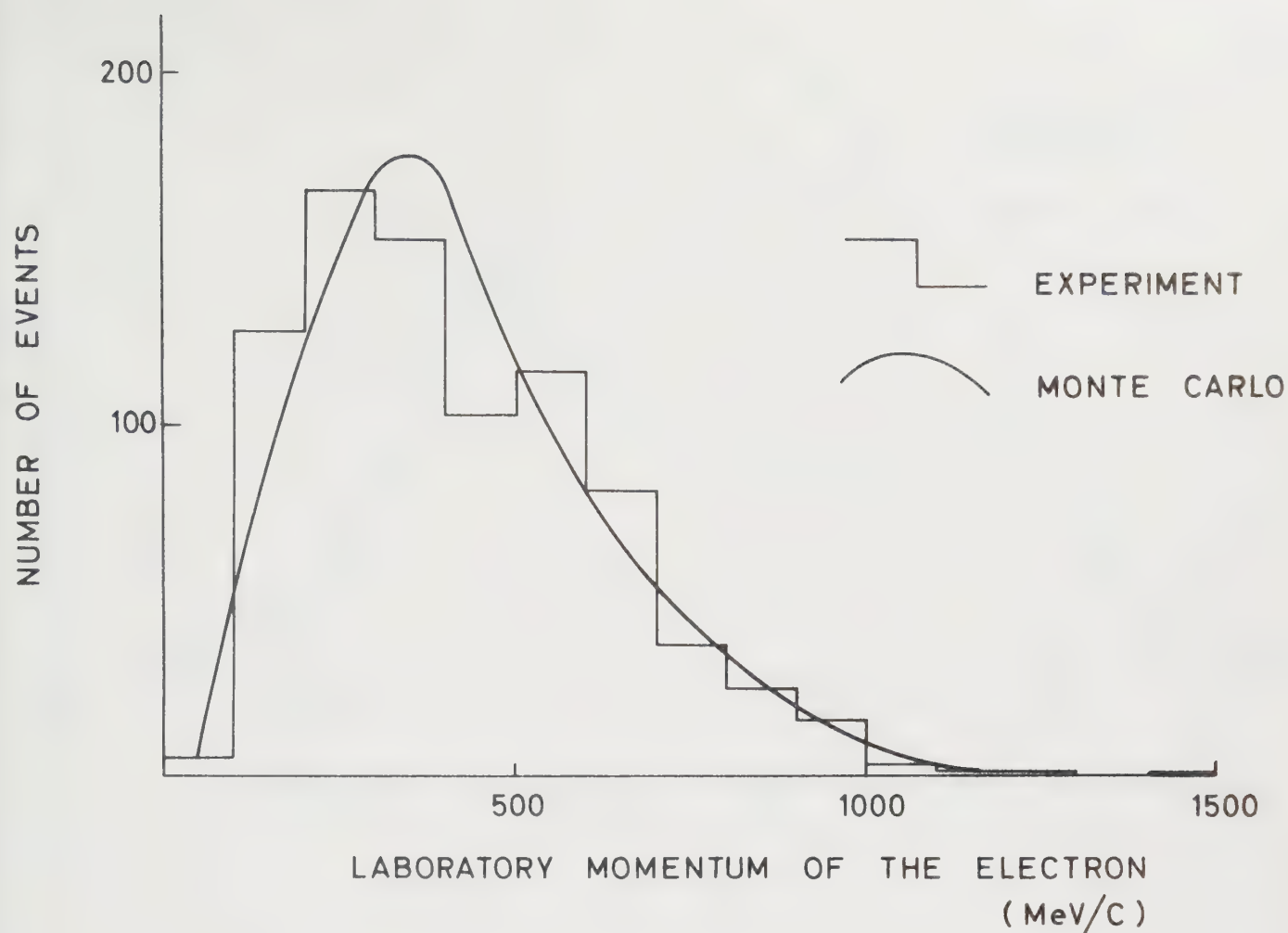


FIGURE 10 COMPARISON OF MONTE CARLO AND EXPERIMENTAL SPECTRA

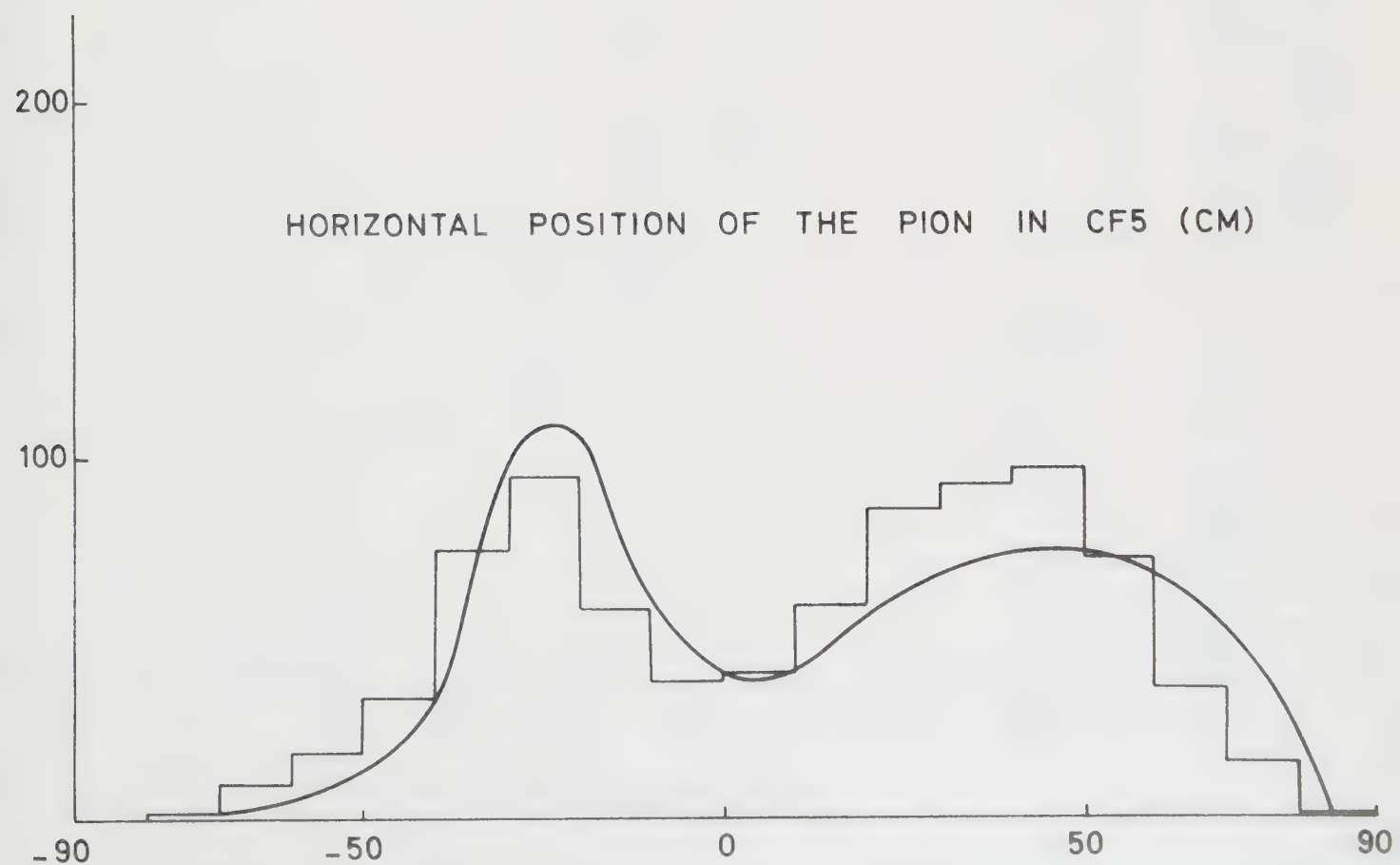
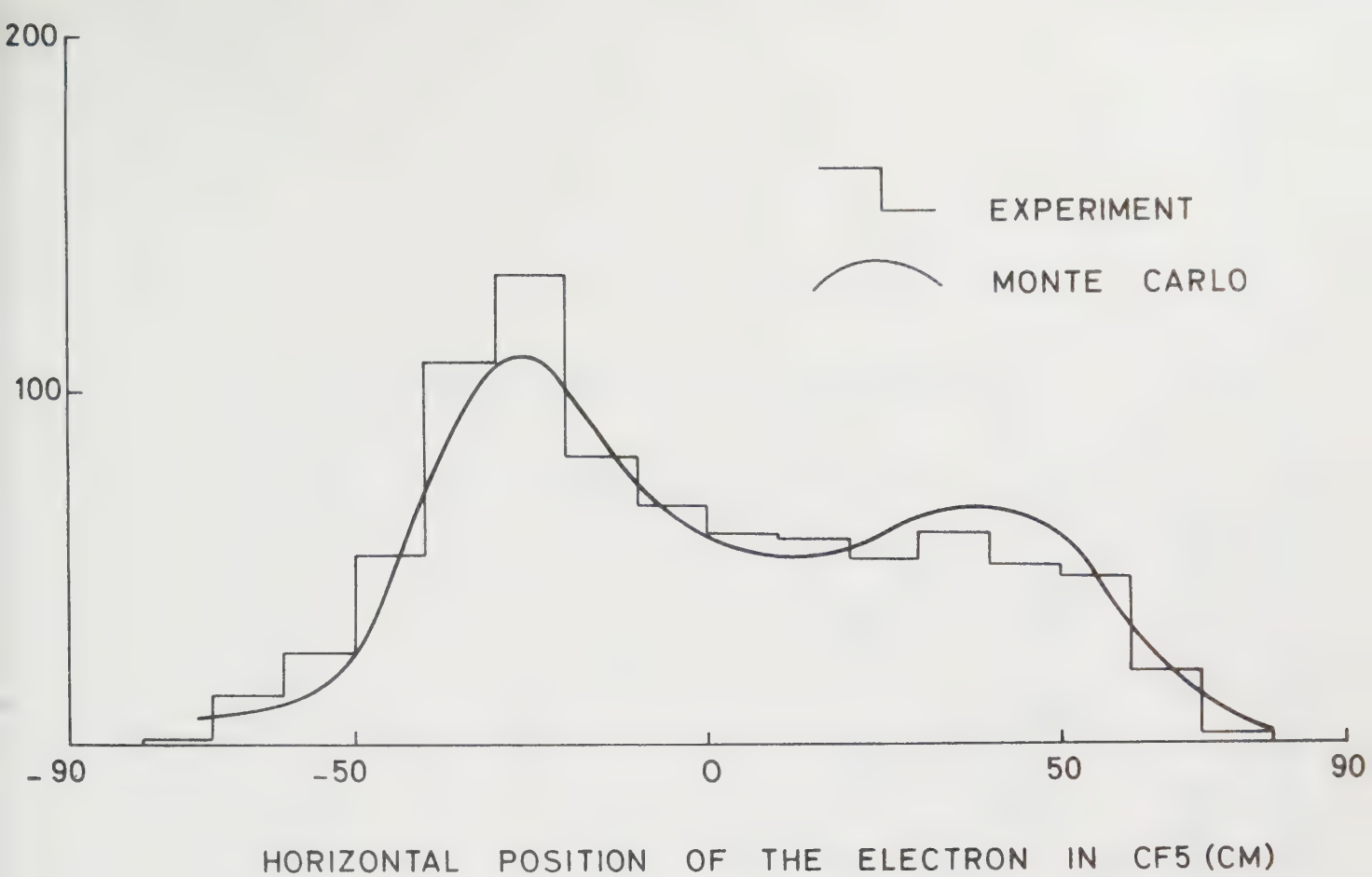


FIGURE 11 COMPARISON OF MONTE CARLO
AND EXPERIMENTAL SPECTRA

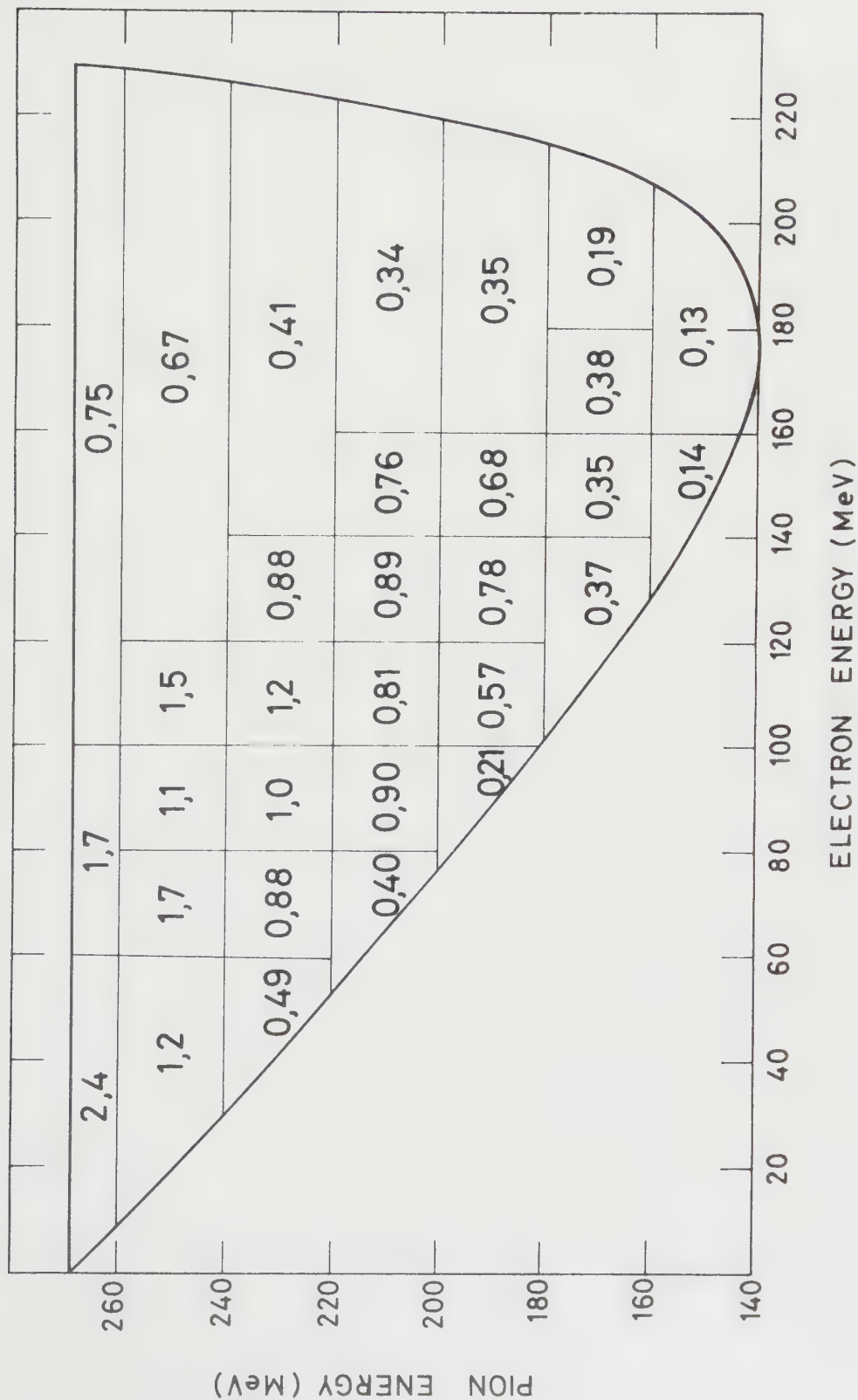


FIGURE 12 MAP OF MONTE CARLO WEIGHTS (arbitrary units)

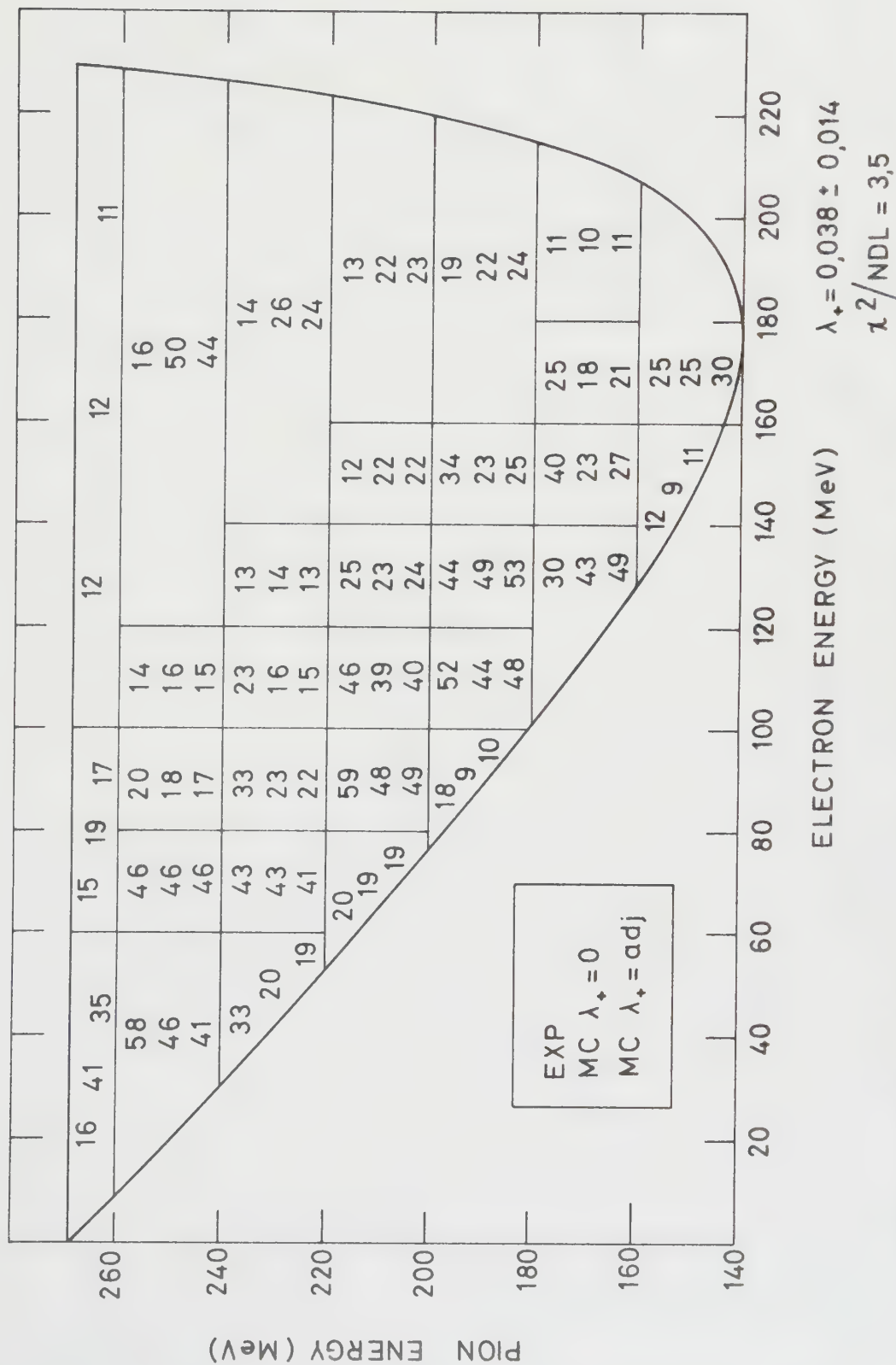


FIGURE 13

CALCULATION OF λ_+ BY MINIMIZATION OF THE χ^2 OVER THE DALITZ PLOT

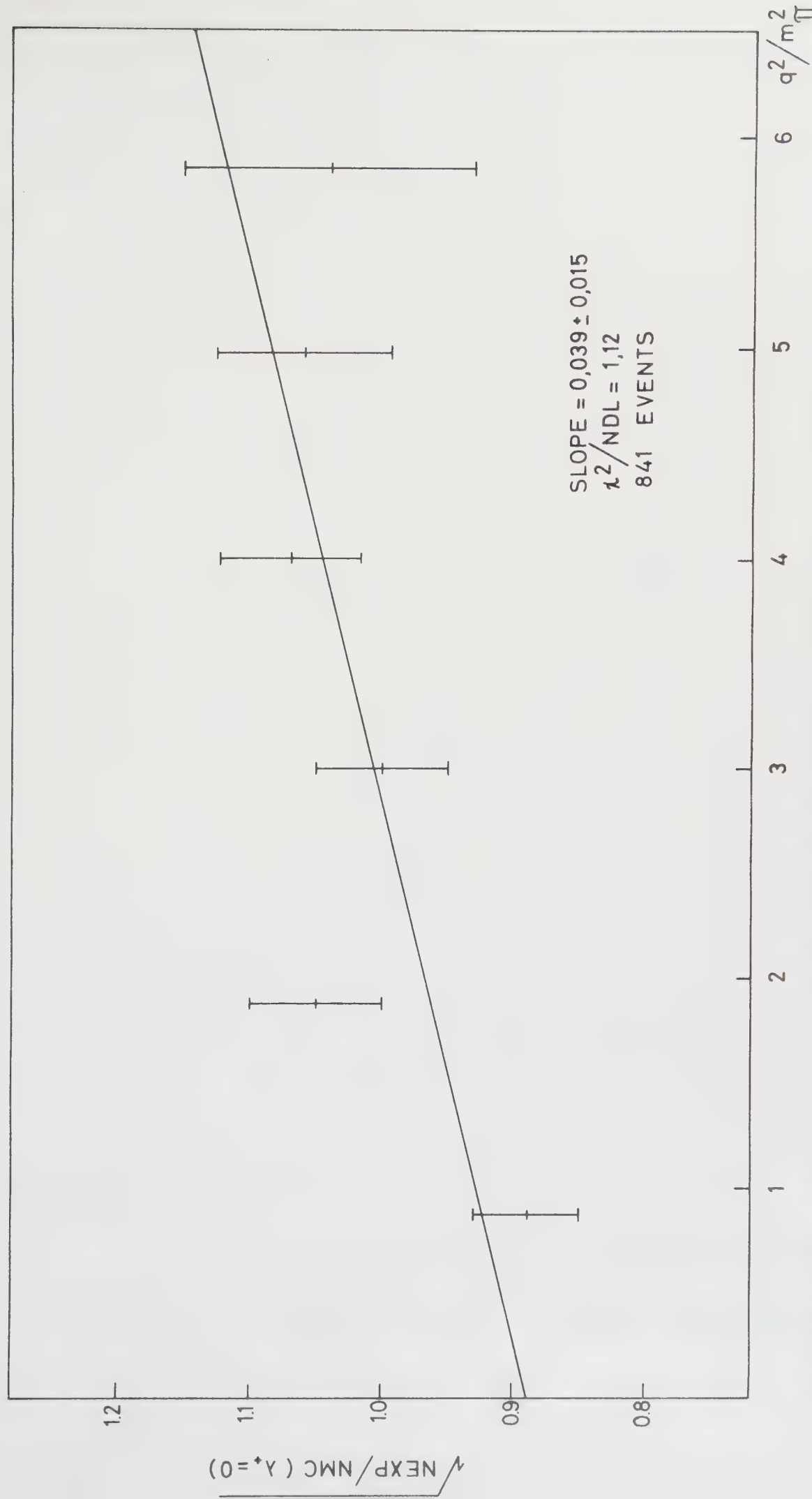


FIGURE 14 CALCULATION OF λ_+ BY THE STRIP METHOD

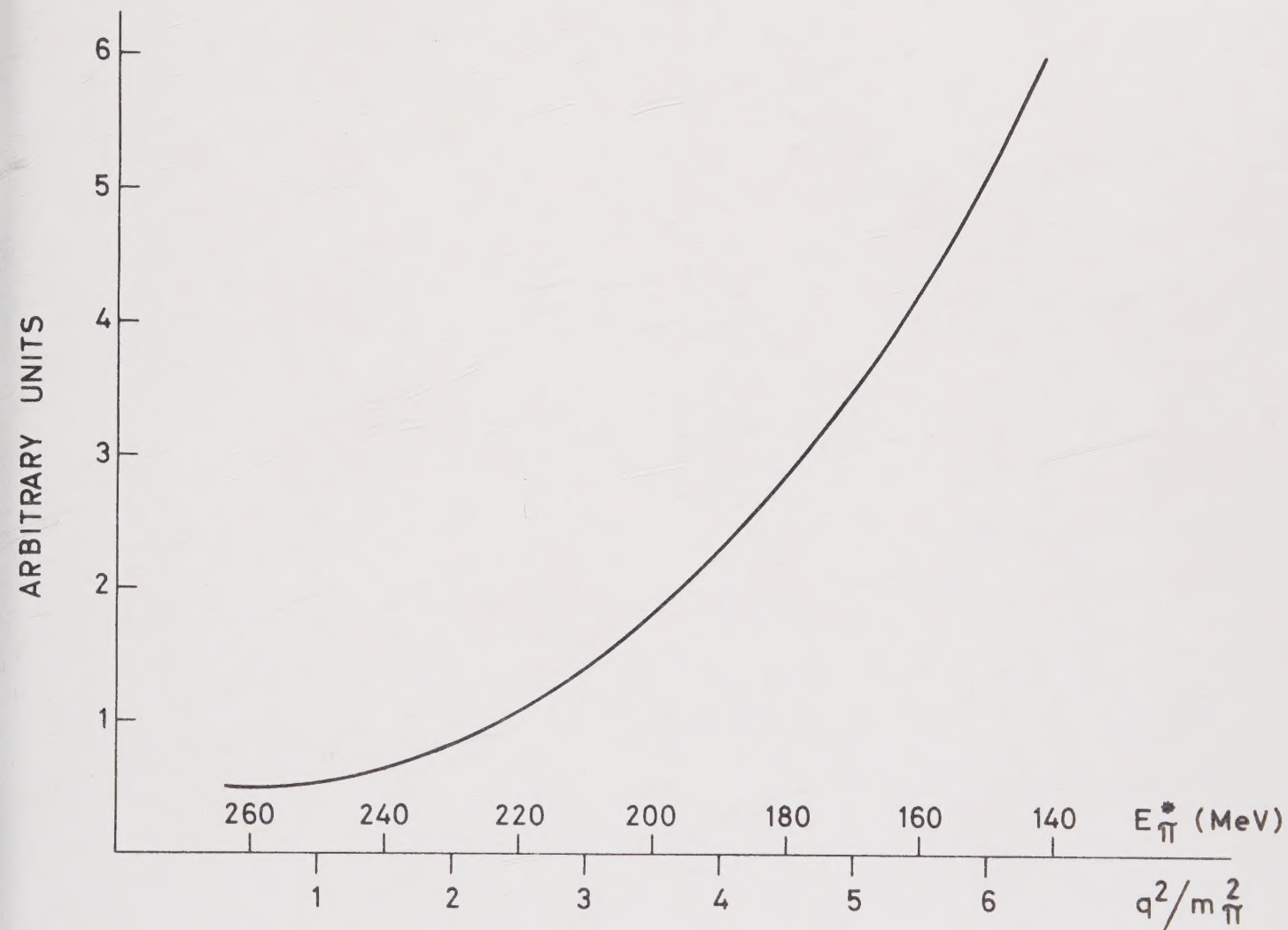


FIGURE 15

DETECTION EFFICIENCY
OF THE SPECTROMETER

